DEVELOPMENT OF A MINIATURIZED LOW NOISE SONAR PRE-AMPLIFIER

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ABSTRACT

This report is concerned with the development of a subminiature SONAR pre-amplifier operating in the 1-20 KC frequency band. The primary requirement is very low circuit noise and major effort is directed to this subject. Several types of input circuits are investigated. An improvement in noise level of 7.5 db over the existing pre-amplifier is attained, together with a 90% reduction in size.

A systematic design procedure to aid in future investigations of this type is developed. The first amplifier stage and the input transformer together determine the ultimate level of circuit noise. Significant improvement over present levels will involve basic research into the noise characteristics of transformers.

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One of the most serious electronic circuit noise problems existing today is in the field of listening Sonar employed by the U.S. Navy. At present the detection range of this equipment, in many cases, is limited by inherent circuit noise. The implications of this restriction, in terms of combat effectiveness, are obvious.

Because Sonar is specialized as to application and frequency range, almost all of the many recent significant developments have been produced by scientists and engineers of the Navy and those in science and industry associated with them. Consequently, only a portion of the vast store of recent knowledge of low-noise circuitry resulting from work in higher frequency applications, such as radar and television, is applicable to Sonar.

One of the consequences of continued improvement of performance of electronic equipment is often a very considerable increase in complexity and size. This is very true in the case of listening Sonar, and to an extent that the installation of this equipment on the already crowded ships of the Navy presents a practical problem of real magnitude. No matter how well a Sonar performs, its value is limited by the restrictions its installation may place on other characteristics of the vessel.

In recognition of the design concept of the "Ship as a whole," the authors have undertaken this study with the dual purpose of reducing the circuit noise level of an existing Sonar equipment, and at the same time, effecting a significant decrease in its physical size.

Purpose of the Investigation

The purposes of this investigation are two:

(1) To develop a SONAR pre-amplifier suitable for subminiature construction with improved low noise characteristics, and

(2) To develop a systematic approach to the general problem of design of SONAR pre-amplifiers, in the hope that it may be of assistance to future investigators in this specialized field.

The Importance of Low Circuit Noise

Inherent noise in an electronic circuit represents a limit on the performance of the circuit. In devices which normally operate with low-level input signals such as radio, radar, and sonar, this limitation can be expressed in terms of reduced detection range. Reduction of inherent noise, i.e., noise generated within the circuit itself, is therefore one means of increasing this range.

Other ways of increasing range are to increase the transmitted power or to change the carrier frequency. In the case of "listening" SONAR equipment, which is our concern here, neither of these means is available. This fact places additional emphasis on the requirement of low circuit noise for the pre-amplifier unit of a listening SONAR:

Compared to the whole broad field of electronics, SONAR is specialized and of limited technical interest to the majority of workers. Its real value, of course, lies in its importance to our national defense. Another factor which somewhat isolates it from the main body of technical effort is its frequency range. The listening SONAR which is the subject of this investigation covers the range from one KC. to twenty KC. Although this range occupies much of the audio frequency band, most audio applications do not suffer the same severe restrictions as to circuit noise. In recent years, the wide use of

carrier transmission in our long distance telephone circuits has left the field of listening SONAR even more alone as an audio-frequency, low-noise application.

Zero Sea State Noise

The ultimate lower limit to the detection ability of listening SONAR is a quantity called "Zero sea state noise". (22) As the term implies, this is the noise level inherent in the sea itself, when the sea is in its quietest state. This noise falls off rapidly with frequency, attaining a very low level at 20 KC. (See curve C, on page 35b.) In order to attain this ultimate level of detection, the inherent noise of the pre-amplifier must everywhere be less than zero sea state noise referred through the transducer.

The Advantage of Miniaturization

In addition to the scientific aspects of this problem, a very real need exists for reducing the weight and space occupied by electronics equipment on Naval vessels. Listening SONAR sets are particular offenders in this respect, in that several dozen identical pre-amplifier units are required, each connecting a line hydrophone to the compensating switch. Successful miniaturization of the pre-amplifier unit, without sacrifice of low-noise characteristics, would therefore be a fruitful means of saving a substantial amount in weight and space aboard each vessel.

A Systematic Approach

From the inception of this work, the authors have attempted to note and record anything which might assist in the future development of equipment of this type, and in particular have searched for relationships between conflicting requirements which might be useful as guiding principles.

In view of the facts that design requirements for listening SONAR are essentially different from many other electronic problems, and, also that these requirements may be expected to change somewhat with each new

design, some of the principles included in this report may at least save time by helping to avoid recourse to cut-and-try methods.

GENERAL PROCEDURE

The function of a SONAR pre-amplifier is to accept a low-level signal at its input, amplify the signal to a more usable level, and pass it on to the next component of the equipment. Of all the various requirements the pre-amplifier is to meet, the most important, by far, is that of low inherent noise. Accordingly, the major emphasis in this work was given to the attainment of minimum circuit noise.

In a multi-stage amplifier circuit, usually the only circuit noise of importance is that generated in the first stage of amplification and any part of the circuit which precedes that stage. This noise and the incoming signal voltage are both amplified by the full gain of the amplifier, and so appear at the output with the same relative magnitudes they had at the input. Noise generated in succeeding stages is amplified only by those stages and so is relatively much less at the output than the noise voltages at the input. This fact focuses major attention on the input circuit design.

In view of the fundamentals discussed above, and considering the additional requirement of miniaturization, the procedure which guided this investigation was as follows:

1. Selection of a low-noise tube for the first stage of amplification. This was based on noise tests of various types of sub-miniature tubes.
2. Design and tests of various circuit configurations for the first stage, using the selected low-noise tube.
3. Selection of a suitable input transformer. This was necessitated by the fact that the best obtainable noise level for the first tube stage was not sufficiently low.
4. Consideration of other requirements - Steps 1, 2, and 3 above were concerned entirely with low noise. Other requirements of less immediate interest to this investigation are frequency

stability. These requirements may be met in a number of conventional ways, but any resulting increase in circuit noise should be minimized.

These steps, and the results of each, are presented in detail in a later section. First, however, a review of some facts about noise itself may be helpful to the reader.

CHAPTER II - BASIC THEORY OF CIRCUIT NOISE

The Nature of Noise

Inherent noise, as commonly applied to amplifier circuits, is defined as an unwanted or spurious signal voltage generated within the amplifier itself. It is random in nature. This investigation is concerned with the level of mean square average noise voltage, in particular its magnitude, its sources, and the reduction of its affects on circuit performance.

Noise Sources

Inherent noise, of importance in the audio frequency range, is produced in several ways:

A.) Thermal Agitation Noise (13)* - The random motion of free elections in a conductor develops small voltages across the conductor in accordance with the following equation:

$$\overline{e}_n^2 = 4kT \int_{f_1}^{f_2} R\,df \qquad \text{(Eqn. 1)}$$

where $\overline{e}_n^2$ = mean squared value of the noise voltage

k = Boltzmann's Constant
= 1.374×10^{-23} joules per degree Kelvin

T = Absolute temperature in °K

R = Real part of the impedance seen at the terminals

f = Frequency

B.) Granular Resistance Noise (1) This noise arises from the fluctuation of contact resistance between adjacent granules of a carbon resistor carrying direct current. The resultant noise voltage is much greater than thermal agitation noise, thus making carbon resistors unsatisfactory for use in low-noise circuits unless they are by-passed by a suitable capacitor.

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* This notation, ( ), references items in the BIBLIOGRAPHY.

C.) Vacuum Tube Noise - There are many sources of noise in a vacuum tube, but the ones of major importance in the audio frequency range are as follows:

1. Shot noise - This is due to a fluctuation of plate current caused by random emission of electrons from the cathode. An indirectly heated cathode is preferable.

2. Reduced shot effect noise - This is a considerably lower value of shot noise caused by the smoothing effect of the space charge on the fluctuations of plate current. Tubes should therefore be operated in the region of space change limited emission. The reduced shot effect noise may be expressed as an equivalent noise resistance by the equation:[12]

$$R_{eq} = \frac{2.5}{Gm} \qquad \text{(Eqn. 2)}$$

where R_{eq} is in ohms.

and Gm is the transconductance in mhos at the operating point.

3. Flicker Effect Noise - This is a relatively low frequency effect caused by local fluctuations of emissivity in the cathode. Present knowledge of the theory of flicker noise is limited, and this subject in under current investigation by van der Ziel.[17]

4. Partition Noise - Partition noise is similar to shot noise but is caused by the division of current between electrodes in tetrodes and pentodes. This current division between plate and screen of a pentode results in a noise level 3 to 5 times as great as that of a triode.[3]

5. Miscellaneous Tube Noises -[23] These noises are due to microphonics, gas content, ballistics, and positive ion current in the grid circuit. They may be reduced to a

negligible level by careful tube design.

D. Noise in Transformers - Theoretically, the equivalent noise voltage of a transformer should be pure thermal agitation noise due to its resistive component. Practically, because of its construction and the material of its core, a transformer is very susceptible to electrostatic and magnetic pick-up. These pick-up voltages can be greatly reduced by proper shielding, but the problem becomes difficult at low frequencies.

In addition there is a source of transformer noise referred to a BARKHAUSEN effect (19). This effect is due to the fact that the magnetization of a ferromagnetic body does not proceed uniformly, but in discrete steps. It has to do with the magnetic domains within the core, and has not yet been fully examined. However, it is caused by a variable or cyclic magnetization. Thus, stray pickup voltages which occur outside the frequency band and are in themselves of no consequence, can induce Barkhausen noise lying in the entire frequency spectrum of the transformer. This noise is likely to be an important factor limiting the sensitivity of high gain audio frequency amplifiers having iron- cored transformers in the input.

In addition to the noise sources above, which are inherent in the individual circuit components, stray pick-up and hum voltages induced anywhere in the circuit must be guarded against. These can be avoided by careful physical design, including lead dress and adequate shielding and, in particular, the use of a d.c. filament supply.

The calculation of the theoretical noise level of an amplifier is relatively simple in the frequency range from one kc to twenty kc. In this range of frequencies, the interelectrode conductances are usually small enough to be neglected, and the components of tube noise voltage and resistor thermal noise may be considered statistically independent.[4] They therefore add quadratically. The calculation procedure consists of applying a simplified form of Equation I for the evaluation of the mean squared effective noise voltage:

$$\overline{e}_n^2 = 4KT\, B_{eq}\, R \qquad \text{(Eqn. 3)}$$

where k and T are as previously defined, and B_{eq} is the equivalent bandwidth (see Appendix pg.48). R is any resistive component of the circuit. In the case of a tube, its equivalent noise resistance may be used as R in Eqn 3. to calculate the equivalent noise voltage.

For a triode, in the frequency band of 1-20 kc:

$$R_{eq} = R_{flicker} + R_{shot}$$

where $R_{flicker}$ and R_{shot} are equivalent resistances due to flicker noise and reduced shot noise. The value of R_{shot} may be accurately predicted by the relationship:

$$R_{shot} = \frac{2.5}{gm}$$

The value of $R_{flicker}$ is frequency dependent and varies with tube type and cathode material. In many cases, however, its magnitude varies inversely with frequency. This noise accounts for the usual low frequency rise in noise spectrum curves.

The frequency at which the flicker noise level equals shot noise cannot be predicted with certainty, generally lying anywhere between 2kc and 10kc or higher for coated cathodes. Gillespie[18] gives the following equation for flicker effect at low frequencies:

$$\frac{\overline{e}_n^2}{df} = \frac{10^{-13}}{f} \qquad \text{(Eqn. 4)}$$

All the noise generated in the amplifier must be referred to the amplifier input, where the comparison of its level with the minimum signal level establishes the amplifier's noise characteristics. In order to refer the noise voltages to the input of the amplifier, noise voltages originating in the plate circuit of the first stage and following stages, are merely divided by the voltage gain between the input and the point in the circuit where the noise voltage is introduced. The total noise voltage referred to the input may then be calculated as follows:[1]

$$\overline{e}_{n_{total}} = \sqrt{\overline{e}_{n_1}^2 + \overline{e}_{n_2}^2 + \overline{e}_{n_3}^2 \pm ----} \qquad \text{(Eqn. 5)}$$

This expression gives noise in terms of a voltage. Equation (3),

$$\overline{e}_n^2 = 4kT\, B_{eq}\, R_{eq} \quad ,$$

enables us to express the noise alternately in terms of an equivalent noise resistance R_{eq} for any component. This is convenient in that it allows direct addition of the equivalent noise resistances and helps to put in evidence the most troublesome components.

Expression of Noise Levels

Noise levels described in this paper are expressed in decibels referred to one volt on a per cycle basis or in equivalent noise resistance. These methods of expression were selected for the twofold purpose of ease of calculation and clarity of meaning in describing the absolute noise level of a circuit. The more familiar

method of expression, noise figure, is avoided since it does not indicate the absolute noise level of a circuit unless the noise power of the source is specified. In many cases, the noise power of the source varies with frequency, thus making the noise figure an unwieldy method of expression.

Method of Noise Measurement

Noise levels were made with the pre-amplifier input shorted and read at the output with a narrow band analyzer. The output reading, at any frequency, is then referred to the input through the total gain between the two points. This method, and equipment used, is described more fully in the Appendix, pg.44.

CHAPTER III

Specific Procedure and Results

In this chapter, the application of the step by step procedure (i.e., tube selection, first stage selection, input transformer selection, and correlation of other amplifier requirements) to the specific design problem is described in detail. The results of the various investigations are presented, forming a basis for the final selection of components and circuits.

In addition to the solution of the specific problem, some general conclusions, where applicable, are drawn. These are based on the theoretical and experimental results of the various investigations and may be of assistance in future work of this type.

Step 1. <u>Selection of Tube Type For The First Stage Of Amplification</u>

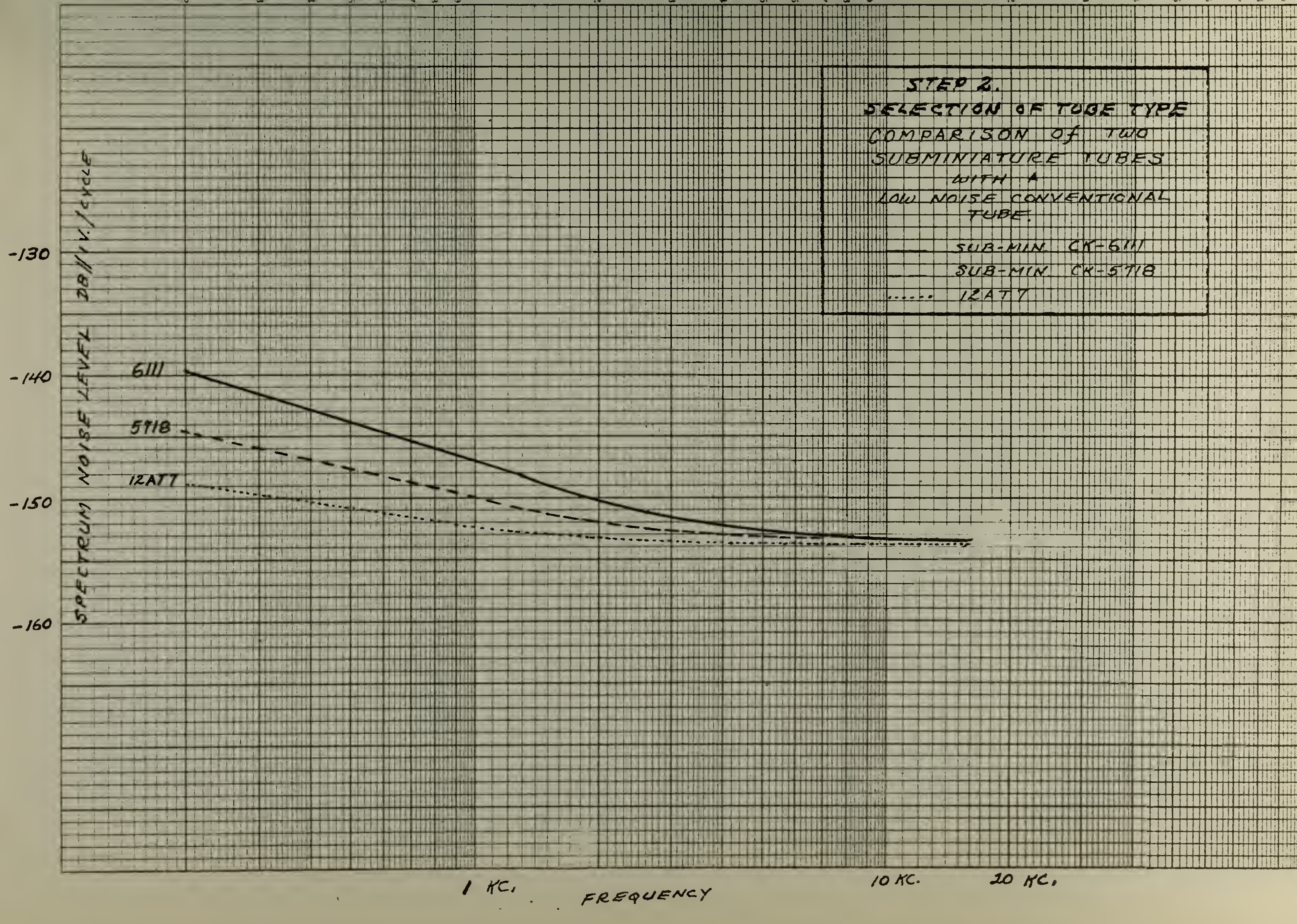
As a guide to tube selection, the equation of North [12] and others provides a fairly accurate basis in the frequency region where reduced shot effect is the major component of tube noise. The high level of partition noise in a pentode then limits the choice to high g_m triodes or triode-connected pentodes.

In the lower frequency range, where flicker noise predominates, there is, to the present knowledge of the authors, no simple guide rule. Cathode materials and cathode sizes directly affect level of flicker noise. However, controlled experiments [4] have indicated a large spread in the magnitude between tubes of the same type as well as in tubes of different types. Reference to the present and future work of van der Ziel may be of assistance in those applications where low frequency noise is critical.

Since the ultimate noise level of the first stage of amplification is set by the tube noise, the selection of the first stage tube type was based almost entirely on noise. Spectrum noise level information was not available for the subminiature tube types which were available, and therefore the noise level of several selected types were compared by measurement. Since flicker noise falls off as $1/f$, shot noise assumes greater relative importance in the upper portion of the frequency band where the noise requirement was most severe. Accordingly, the equation $R_{shot} = \frac{2.5}{g_m}$, served as a guide in determining which tube types were to be measured.

Based on this criterion, the noise levels of several high g_m types were measured by the method outlined in the Appendix on pg. 44. The results, shown on pg. 15a, represent the average readings of several tubes of the same type, and are compared with a low noise regular triode (12AT7). As can be seen from the plot, the difference in noise levels at the high end of the band is of the order of only 1/2 db. Therefore, the type 6111 double triode was selected for use in the first stage, based on its low noise level and the provision of an extra tube in a single envelope, which might later be useful in the miniaturization aspect.

A further reuirement which must be kept in mind during the selection of tube type is that the amplification factor of the tube must be sufficiently high to achieve effective reduction of the noise of succeeding stages to a level which is negligible relative to the first stage.


STEP 2.
SELECTION OF TUBE TYPE
COMPARISON OF TWO
SUBMINIATURE TUBES
WITH A
LOW NOISE CONVENTIONAL
TUBE.
SUB-MIN CK-6111
SUB-MIN CK-5718
12AT7
6111
5718
12AT7
-130
-140
-150
-160
SPECTRUM NOISE LEVEL DB//1V./CYCLE
1 KC.
10 KC.
20 KC.
FREQUENCY

Step 2. Design of the First Stage of Amplification

Having selected a tube of suitably low noise characteristics, the design of the rest of the first stage is the next logical step. The tube itself is only one source of noise, and, of course, requires for its operation some plate load resistance and cathode bias resistance. Any resistance in its plate, cathode, or grid circuit is a noise source and as such, must be considered with tube noise in the stage design.

These are, however, a number of fundamentals which apply generally to this problem. They are:

a.) The tube should be operated well within the region of space-charge limited emission in order to obtain the reduction in shot noise effected by the smoothing action of the space charge.

b.) Sufficient bias should be supplied to prevent excessive noise due to grid current flow.

c.) Wire wound resistors, rather than carbon, should be used in the current carrying circuits of the first stage, except where the resistor is bypassed by a suitably large capacitor. This is required because of the high noise level due to variation of contact resistance in carbon resistors.

d.) D-c filaments and well filtered plate supply provide an effective method of reducing hum.

e.) Adequate shielding of signal leads in the input circuit and care in the wiring layout is required to avoid excessive pickup.

Obviously, the values of resistance associated with the tube stage cannot be selected solely on the basis of noise. It can be stated, however, that if the stage gain is moderately high, the first stage load resistance, as well as second stage tube noise, may be reduced to a negligible amount because the equivalent noise resistance is reduced by the square of the voltage gain in referring it to the grid, or input. On the other hand, un-bypassed cathode resistance, which may be desirable for the application of local inverse feedback to the stage, may constitute a significant source of noise.

With these general principles kept in mind, the noise level of a number of circuit types was investigated in order

to provide a basis for the design of the first stage. The results of these investigations are described in the following pages.

a.) <u>Noise Level of a Single Triode Stage with Cathode Degeneration.</u>

The first circuit which was investigated is shown in Fig. 1.

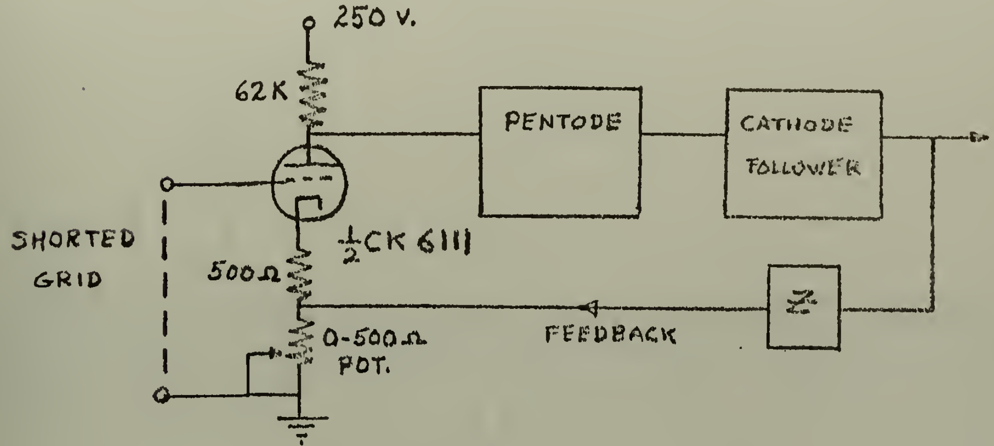


Fig. 1 . Single Triode Test Circuit

This simple stage provides a voltage gain of 20 db, and has an equivalent input noise level as shown on pg.17a. This noise level is used as a reference with which the various other circuits are compared.

b.) <u>Effect of Un-Bypassed Resistance in the Cathode Circuit</u>

It is to be expected that any resistance in the grid to cathode circuit would generate a thermal noise in this circuit. The noise measurements of the circuits of Figs. 1 and 2 ,shown on pg.17a, indicate this clearly.

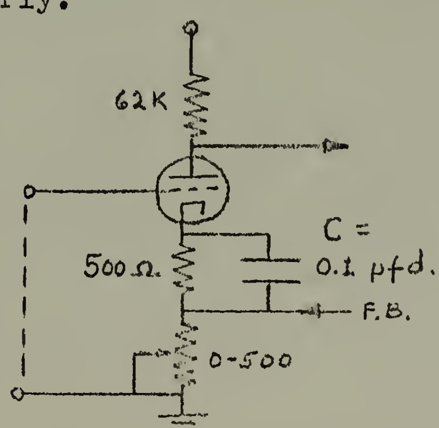
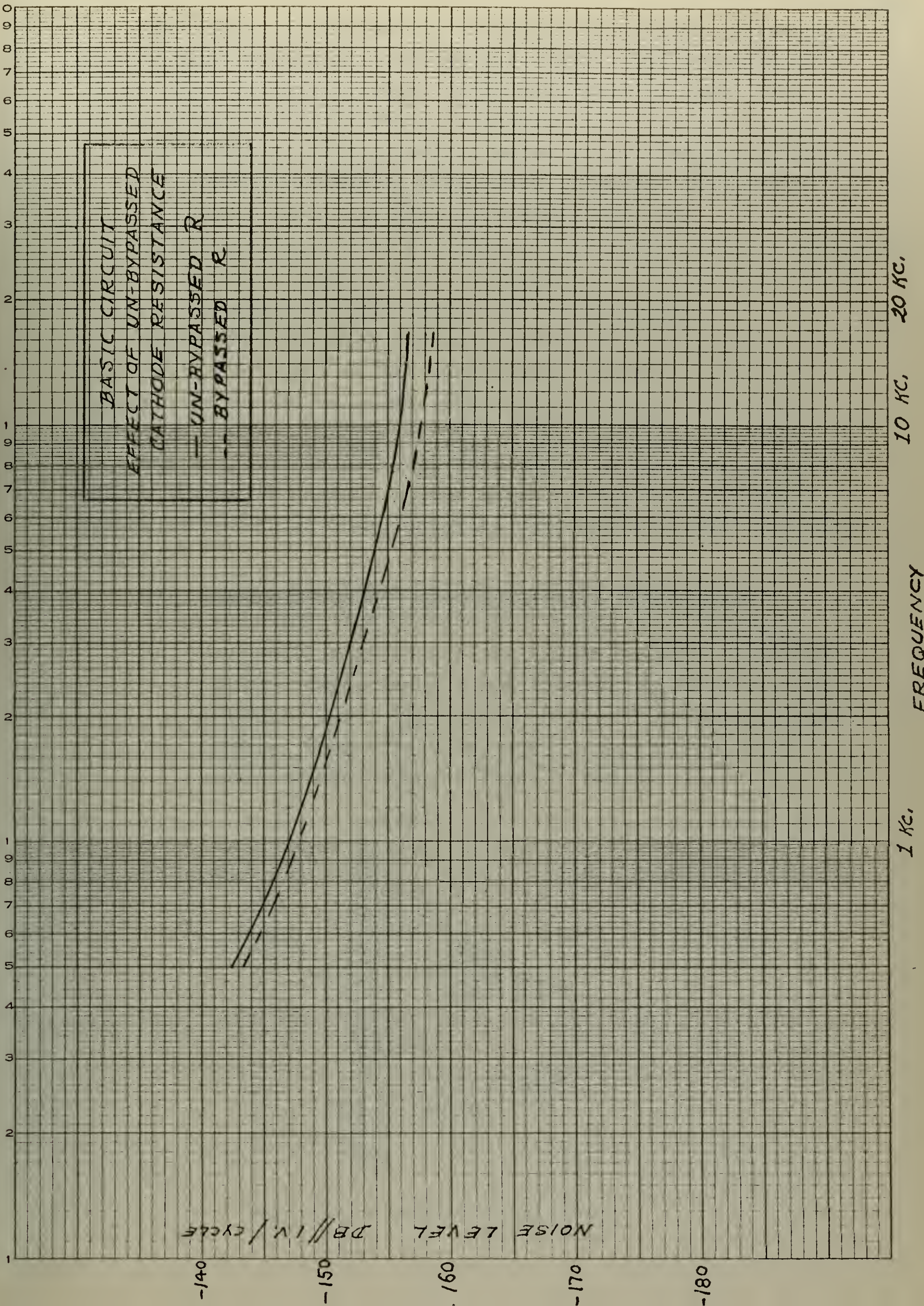


Fig. 2. Single Triode With By-Passed Cathode Resistor


BASIC CIRCUIT
EFFECT OF UN-BYPASSED
CATHODE RESISTANCE
— UN-BYPASSED R
-- BYPASSED R
FREQUENCY
1 KC,
10 KC,
20 KC,
NOISE LEVEL DB//1 V./CYCLE
-140
-150
160
-170
-180

The effect of by-passing the 500 ohms cathode resistance results in a reduction of noise level of approximately 2 db at the high end of the band. This reduction in noise level is the result of two effects, namely:

1.) Shunting of the thermal noise of the 500 ohm cathode resistor, thereby effectively removing this noise source from the grid-to-cathode circuit.
2.) The reduction of the effect of the noise in the succeeding stages caused by the increased gain of the first stage when the cathode resistance is by-passed.

The manner in which the noise level of the by-passed circuit approaches that of the unby-passed circuit at the lower frequencies should be noted. This may be explained by the reduction in the shunting effect of the capacitor at lower frequencies together with the fact that, in this frequency range, the major portion of the noise is due to the flicker effect of the first tube, which is unaffected by the capacitor action.

c.) <u>Use of Two Tubes in Parallel For First Stage</u>

Since the reduced shot noise of a triode is inversely proportional to g_m the parallel operation of two tubes, doubles the value of g_m over that of a single tube, and should therefore reduce the noise level. This effect is shown by the noise level measurements taken of Figs. 1 . and 3.

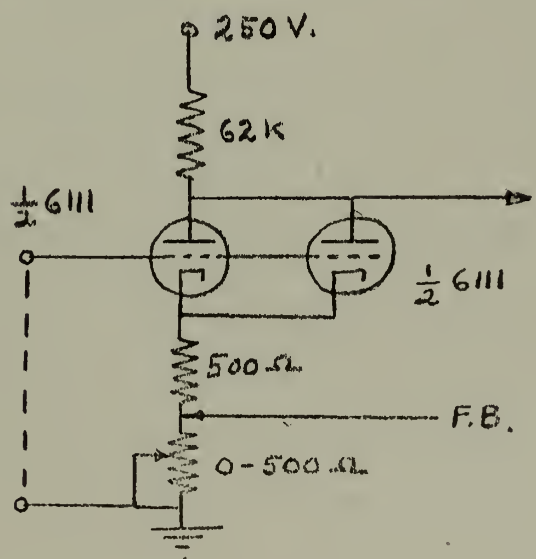
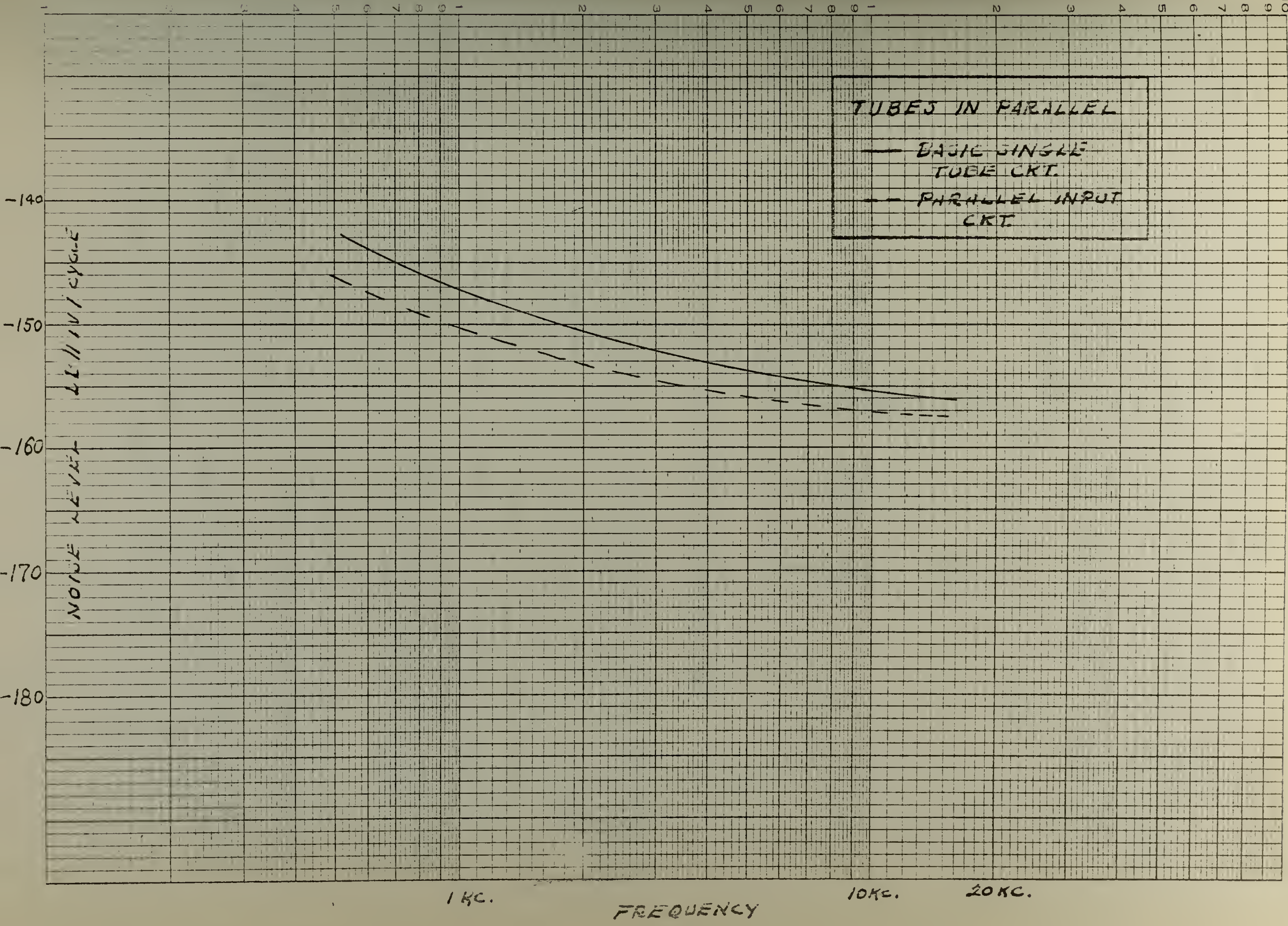


Fig. 3. Parallel Input Stage

These measurements, shown on pg.19a, show a reduction in noise level of approximately two db under the circuit conditions. The amount of reduction will of course depend upon the relative proportions of those noise components which show the $\frac{1}{g_m}$ dependence and those noise components which are independent of g_m.


TUBES IN PARALLEL
BASIC SINGLE TUBE CKT.
PARALLEL INPUT CKT.
NOISE LEVEL DB/1V/CYCLE
-140
-150
-160
-170
-180
1 KC.
10 KC.
20 KC.
FREQUENCY

d.) Cascode Circuit Investigation

The cascode circuit, shown in its basic form in Fig. 4, was the result of the work of Wallman and others in attempts to reduce the noise figure of high frequency amplifiers. Since then, the cascode has been used with considerable success in reducing the noise level of radar, television, and other high frequency circuits.

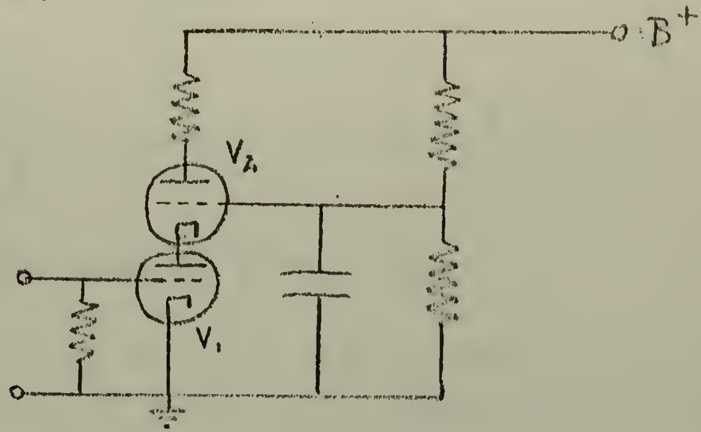


Fig. 4. The Basic Cascode Circuit

The reduction in noise level when compared with the straight pentode amplifiers used in high frequency applications is achieved in the following manner. The grounded grid stage, V_2, tends to stabilize the plate voltage of the lower tube, but still allows its plate current to pass through the load resistor. If complete stabilization of the plate potential of V_1 were attained, a signal e_1, on the grid of V_1, would cause a plate current, $g_m e_1$ to flow through both tubes and the load resistor. Thus the voltage gain of the cascode circuit would approach $g_m R_2$, which is the voltage gain of a pentode.

In this manner, the cascode achieves the high gain of a pentode stage without the high noise level associated with the partition noise of a pentode.

The purpose of this investigation was to compare the noise level of the cascode circuit with that of a low noise triode circuit for audio frequency application. The test circuit used to accomplish this is shown in Fig. 5.

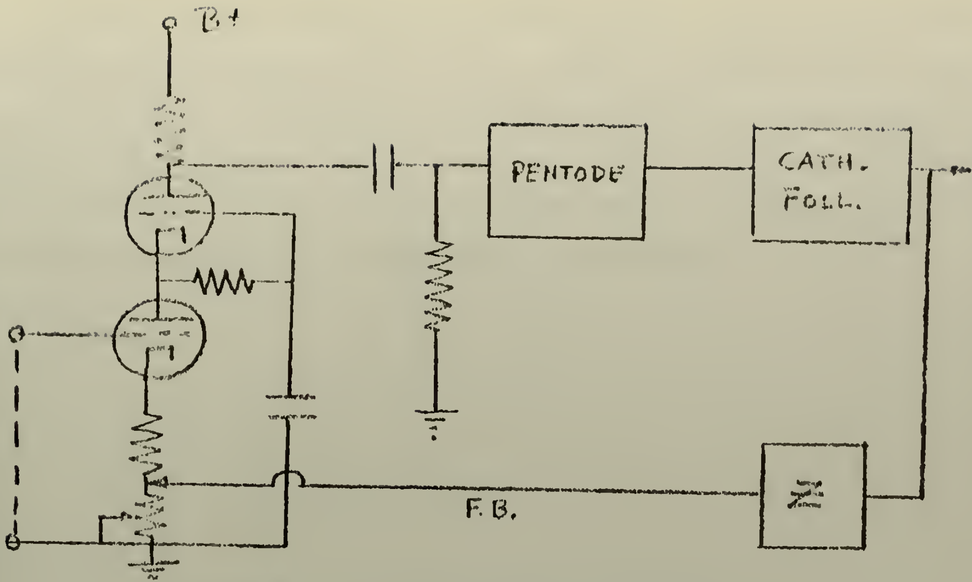


Fig. 5. Cascode Test Circuit

The measured gain of the cascode circuit was 31 dbv, as compared with a gain of 20 dbv in a similar single triode circuit. The resultant noise level of the cascode is compared with that of the single triode stage on pg.27a. The results show a negligible difference in noise level.

As shown in the Appendix on pg.54, the equivalent noise level at the input of a cascode amplifier is made up of the following components:

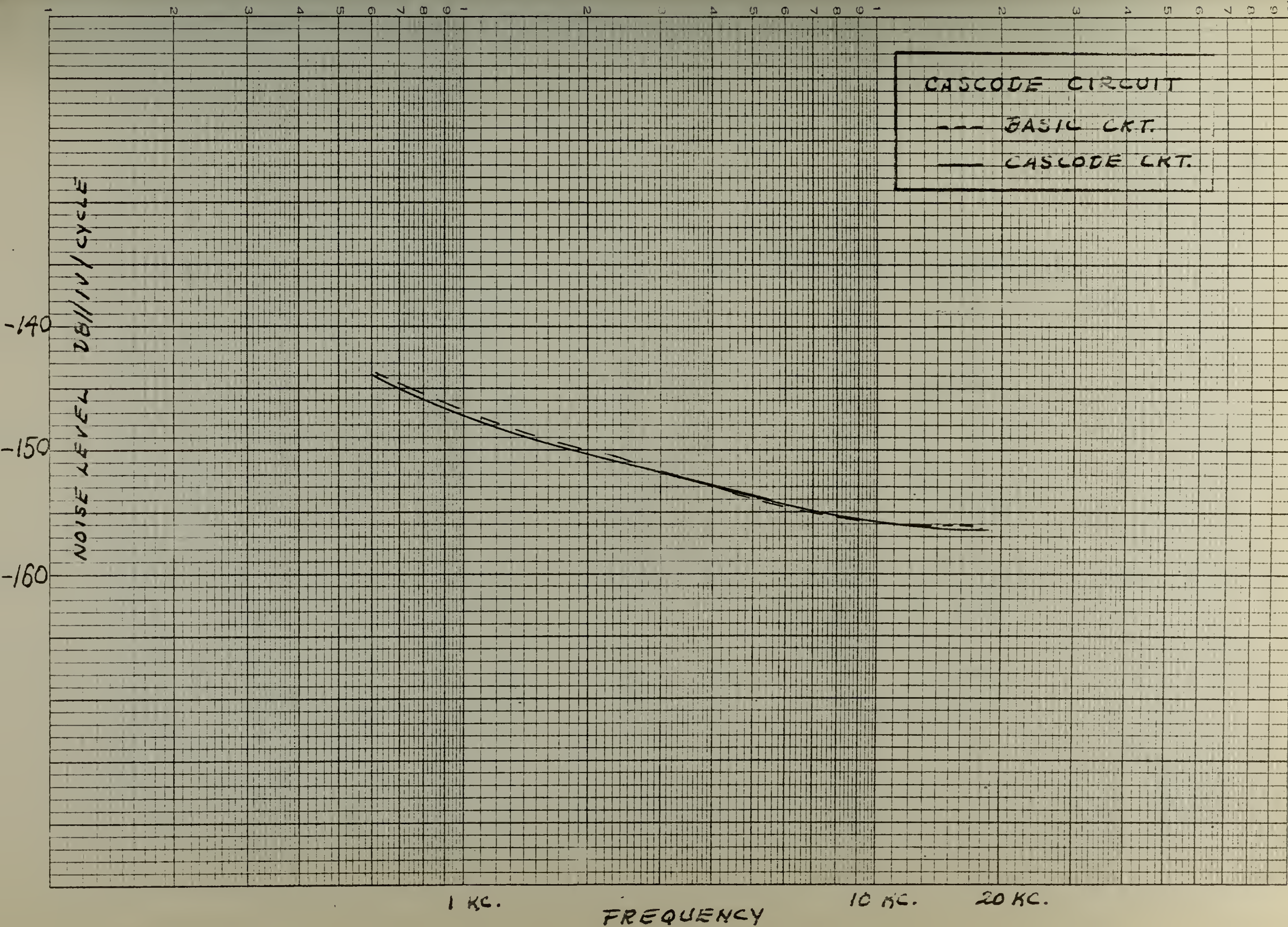
e_{n1} = noise of the lower triode stage, V_1.

$\frac{e_{n2}}{\mu_1}$ = noise of the upper triode, V_2, referred to the input.

$\frac{e_{n3}}{G_1}$ = noise of the succeeding stage referred through the stage gain, G_1, to the input.

Of these three components, e_{n1} has the greatest magnitude by far, since e_{n2}, the upper triode noise, is reduced by the factor $\frac{1}{\mu_1}$, and e_{n3} is similarly reduced by the factor $\frac{1}{G_1}$, where G_1 approaches the high gain of a pentode stage. Thus the equivalent noise level of the cascode circuit is essentially equal to e_{n1}, which is, of course, the equivalent noise level of a high gain triode circuit.

Based on the preceding theoretical and experimental results, the authors conclude that the cascode circuit does not offer any significant reduction in noise level in the audio frequency range when compared with a triode stage, providing the latter has sufficient gain to reduce the effect of the noise of the following stages to a negligible amount.


CASCODE CIRCUIT
--- BASIC CKT.
—— CASCODE CKT.
NOISE LEVEL DB//V/CYCLE
-140
-150
-160
1 KC.
10 KC.
20 KC.
FREQUENCY

e.) Effect of Feedback on Noise

Since the application of inverse feedback provides an effective method of achieving desired gain stability in the amplifier, the effect of feedback on the noise level was investigated. The test circuit was as shown in Fig. 6.

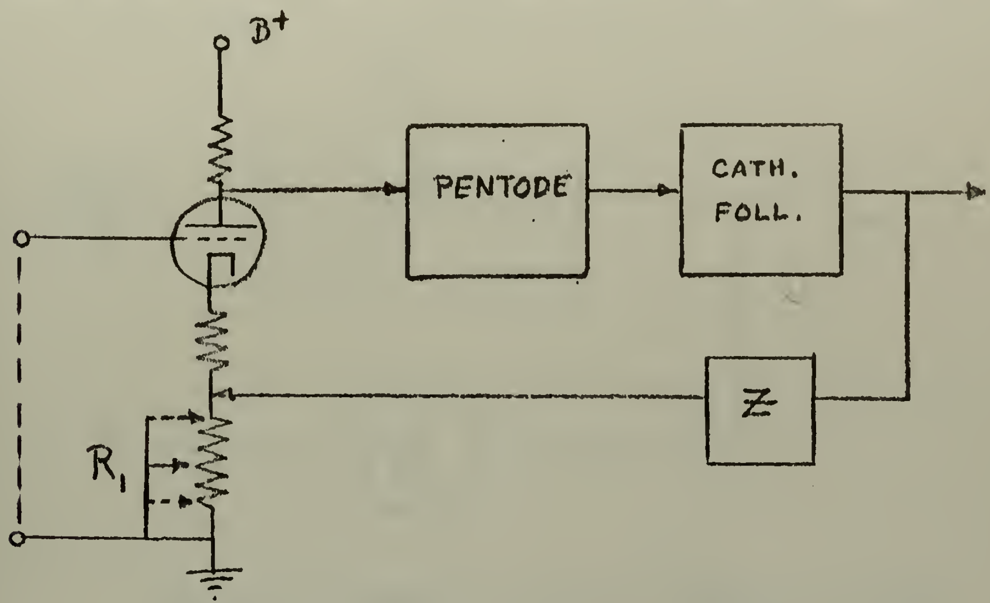


Fig. 6. Test Circuit For Feedback Effect

The noise level was measured for various settings of the feedback potentiometer, R_1, with the results as shown on pg. 25a . The results indicate a small increase in noise level for increase in negative feedback.

Theoretically, inverse feedback will reduce the level of noise at the output of an amplifier in the same ratio that the signal is reduced (providing stage gains are unchanged), thus causing the signal to noise ratio to remain constant. Based on this, we could expect that the equivalent absolute level of noise in the input circuit would remain unchanged, provided that the resistive components of the input circuit remain unchanged. As can be seen from the test circuit, the resistance in the cathode circuit increased with increased

feedback, thus increasing the noise level by a slight amount.

For the test circuit in question, there was an additional effect on the noise level, due to the reduction in gain of the first stage. This reduction in gain was caused by the degenerative effect of the additional un-bypassed resistance in the cathode circuit. This reduction in first stage gain allows the second stage noise to become more effective. This effect may be illustrated by the block diagram of Fig. 7 .

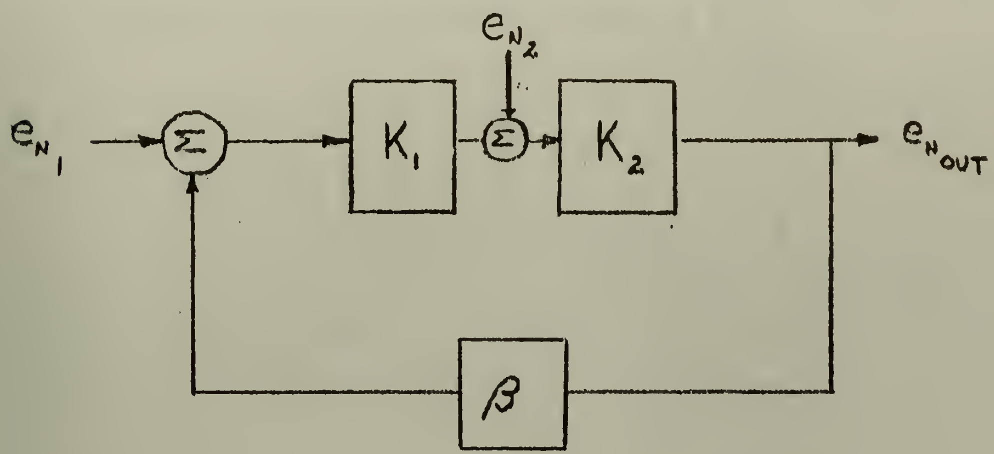


Fig. 7. Block Diagram of Feedback Circuit

In Fig. 7 , e_{n1} and e_{n2} represent the noise voltage introduced at the first and second stage inputs respectively. Without feedback the mean squared noise voltage at the output is as follows:

$$(e_n)^2_{out} = (K_1K_2e_{n1})^2 + (K_2e_{n2})^2$$

To refer the output noise to the input, we divide by the voltage gain squared, $(K_1K_2)^2$.

$$(e_{equiv.})^2 = \frac{(K_1K_2e_{n1})^2 + (K_2e_{n2})^2}{(K_1K_2)^2} = e_{n1}^2 + \frac{e_{n2}^2}{K_1^2}$$

Now with feedback applied, together with a reduction in gain of the first stage as occurred in the test circuit, the noise level at the output becomes:

$$(e_n)^2_{out} = \left[\frac{K_1' K_2 e_{n1}}{1-BK_1'K_2}\right]^2 + \left[\frac{K_2 e_{n2}}{1-BK_1'K_2}\right]^2$$

Where $K_1' < K_1$.

To refer this noise voltage to the input, we divide by the square of the closed loop gain, $\left[\frac{K_1'K_2}{1-BK_1'K_2}\right]^2$

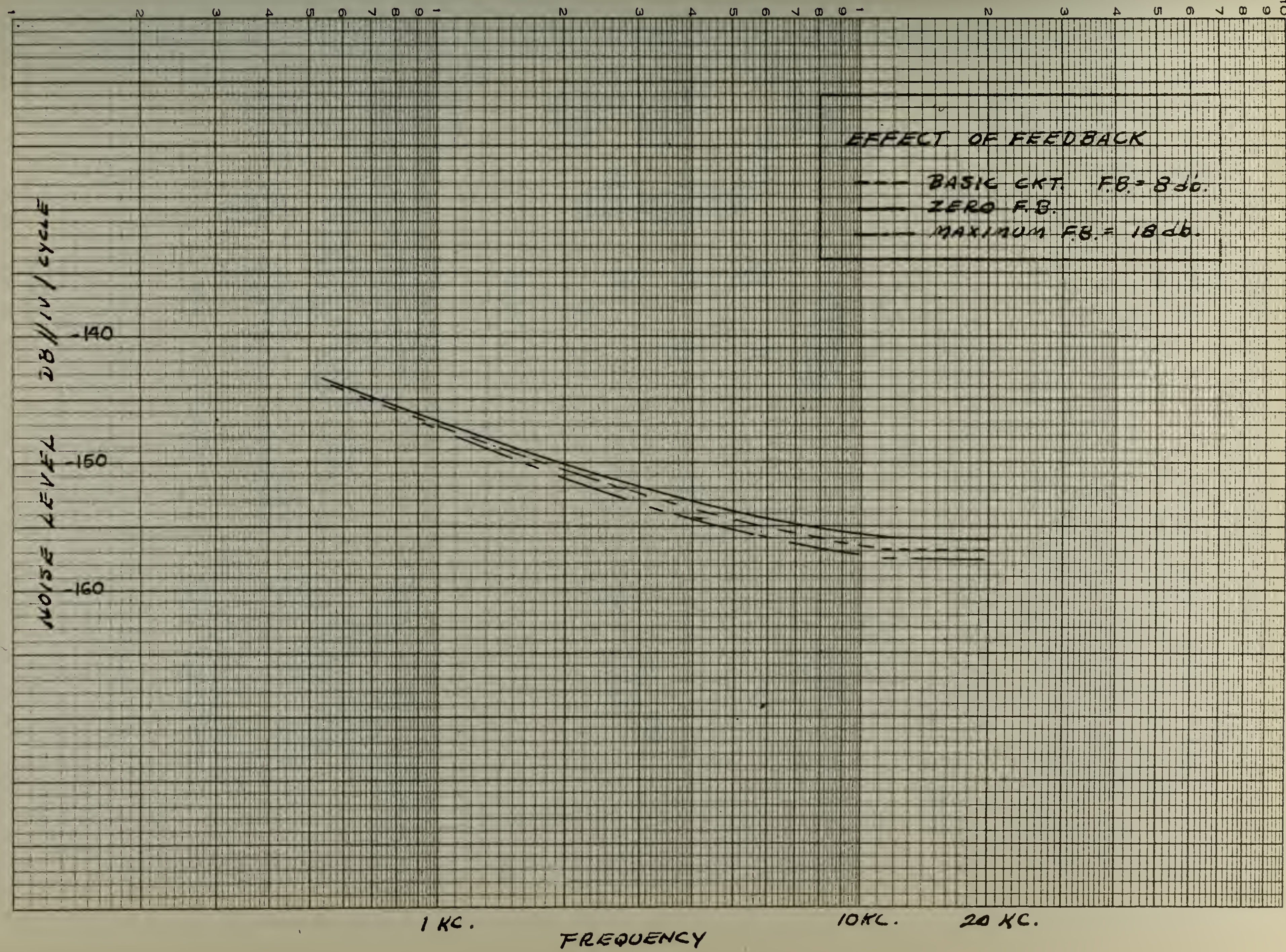
$$(e_{equiv})^2 = \frac{\dfrac{(K_1'K_2e_{n1})^2 + (K_2e_{n2})^2}{(1-BK_1'K_2)^2}}{\dfrac{(K_1'K_2)^2}{(1-BK_1'K_2)^2}}$$

$$(e_{equiv})^2 = e_{n1}^2 + \frac{e_{n2}^2}{(K_1')^2}$$

Thus, the equivalent noise level of the case with feedback is seen to be greater than in the case without feedback, due to the reduction in first stage gain from K_1 to K_1'.

In the test circuit employed, the rise in noise level, due to the effects of increased cathode resistance and reduction in gain of first stage when feedback was applied, was small enough to be considered negligible. In many applications, however, this is not the case, and a significant rise in noise level may result from these effects.

In conclusion, it might be stated that while feedback itself does not affect the equivalent input noise level, the changes in circuitry required to achieve the feedback are capable of increasing the noise level.


EFFECT OF FEEDBACK
BASIC CKT. F.B. = 8 db.
ZERO F.B.
MAXIMUM F.B. = 18 db.
NOISE LEVEL DB//1V/CYCLE
-140
-150
-160
1 KC.
10 KC.
20 KC.
FREQUENCY

(f.) TRANSISTOR NOISE INVESTIGATION

Until recently, the noise factors of junction type traisistors were so high as to render them useless for low noise applications. The developments of the past two years have resulted in a substantial decrease in the noise level of transistors, The purpose of this investigation is to compare the noise level of a transistor stage with a triode stage of comparable gain.

Transistor

The transistor circuit is as shown in Fig. 8.

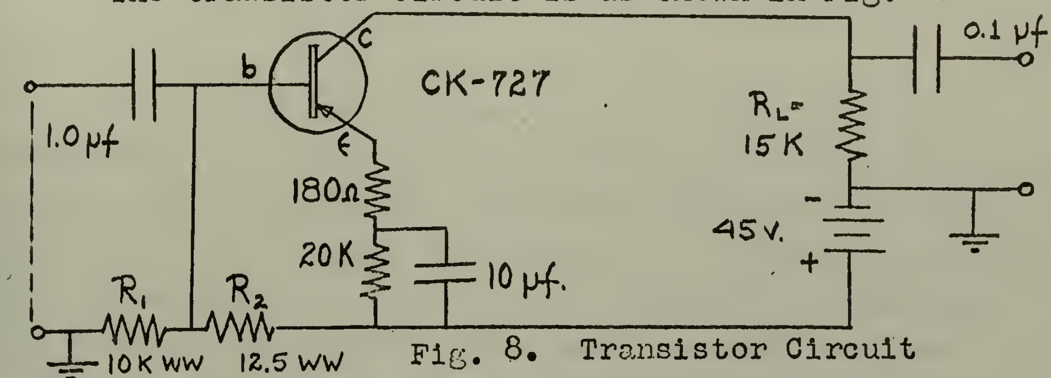


Fig. 8. Transistor Circuit

The CK-727 in this circuit yielded an average voltage gain of 35.5 db for a low input impedance (C=1.0 ufd). The average alpha of the transistors used was 0.97. Furthermore,the transistors were selected for low noise level.

Shea [20] recommends operation with a low collector to emitter voltage in order to obtain the low noise feature. In view of this, the circuit was adjusted so the V_{ce} was less than two volts.

Results

Noise spectrum measurements were taken for three different circuits, with results as shown on pg.27a . Each of these curves represents the average level of three transistors tried in each circuit.

Curve (1) represents the average noise level for the circuit of Fig. 8 , with R_1 = 10K, R_2 = 12.5K, and C = 1.0 ufd.

Curve (2) represents the level for a 10-times

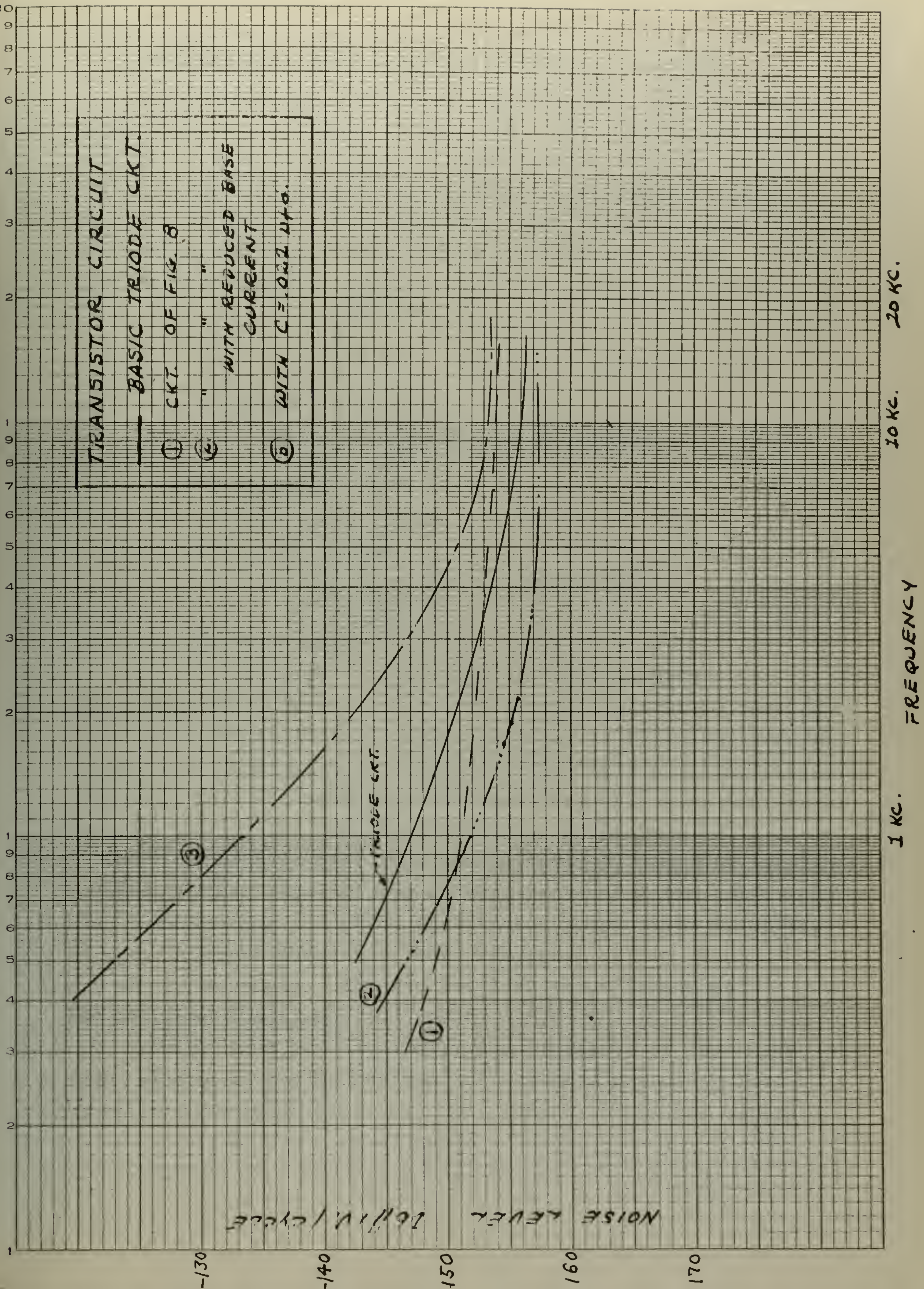
increase in both R_1 and R_2, in order to observe the effect of reduced base current. Curve (3) gives the level with R_1 = 100K, R_2 = 125K, but with a source capacitance of 0.022 ufd.

Both curves (1) and (2) indicate the relationship in the frequency range below 1kc as reported by other authors [20]. They also show an increase in noise level with decrease in base current.

The large increase in noise level, particularly at low frequencies, indicates the significant effect of source impedance on noise level. [21]

The main conclusion to be drawn from this brief investigation is that the noise level of a transistor circuit, with a low input impedance, may approach the noise level of a vacuum tube circuit. The effects of the input impedance on noise level, together with the fact that selected transistor units must be used, are disadvantages which might prevent their use in a low noise sonar application at this time.

In view of the rate of recent development in this field, however, the use of transistors for low noise applications is a definite possibility for the future.



TRANSISTOR CIRCUIT
BASIC TRIODE CKT.
① CKT. OF FIG. 8
② " " WITH REDUCED BASE CURRENT
③ " " WITH C=.022 μfd.
TRIODE CKT.
FREQUENCY
1 KC.
10 KC.
20 KC.
NOISE LEVEL DBV/CYCLE
-130
-140
150
160
170

Various First Stage Circuits - Summary of Results

The significant results of the foregoing sections are summarized on the curves of page 28a. The circuit selected for use as the first stage of amplification was the single triode stage with partial by-passing of the cathode resistor as shown in Fig. 9. The resultant noise level was 158.7 db//1 volt per cycle at the high end of the frequency band. Of the vacuum tube circuits, only the dual input circuit exhibited a

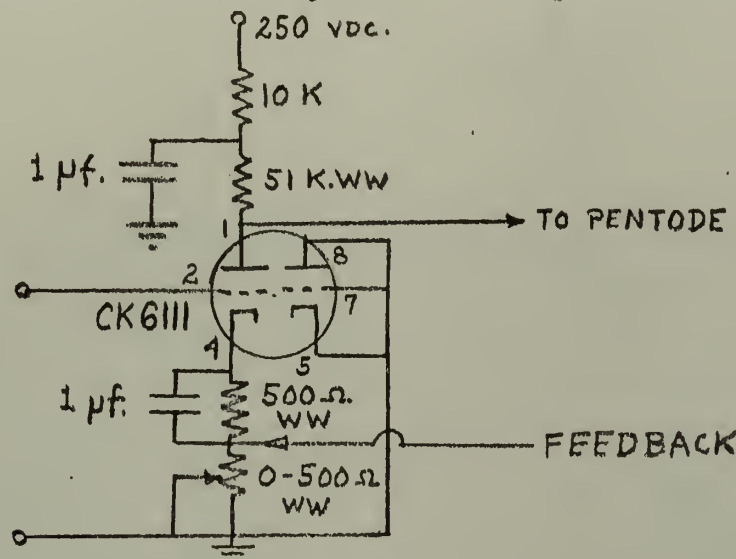


Fig. 9. First Stage of Amplification

comparable noise level, and we may expect a slightly lower level from this circuit if the cathode resistor is bypassed. However, the dual input stage was rejected on the basis that the slight reduction in noise level was not worth the use of an additional tube in view of the miniaturization requirements.

The transistor circuit compares favorably but was not investigated further for this application because of the following:

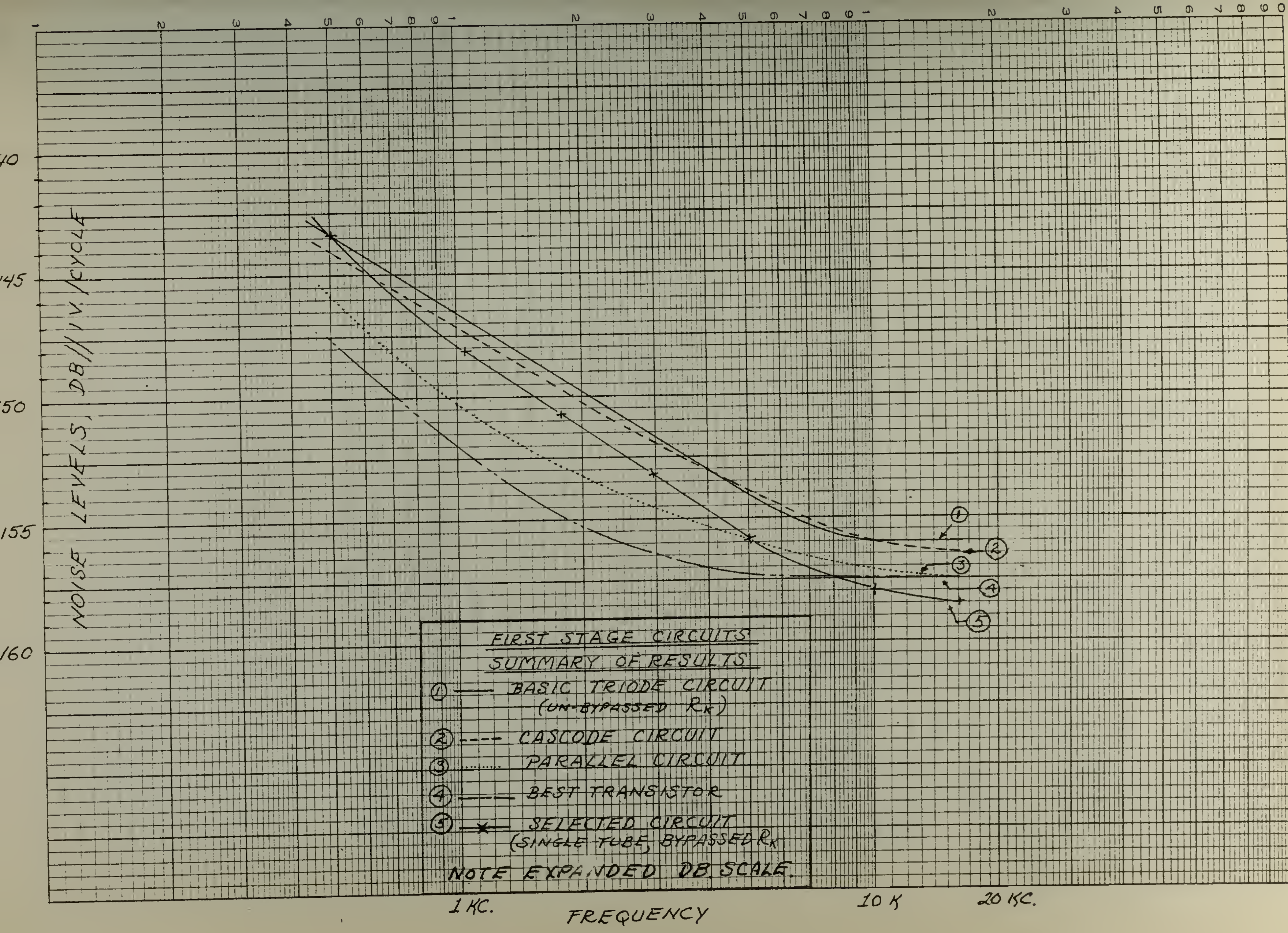
1.) Careful selection of transistors would be required.

2.) Temperature stability problems.

3.) Dependence of noise level on input impedance.

The Required Noise Level

Thus far, we have discussed the necessity of low noise with respect to the noise level of the sea, without being specific as to what this level is. Noise in the sea exists



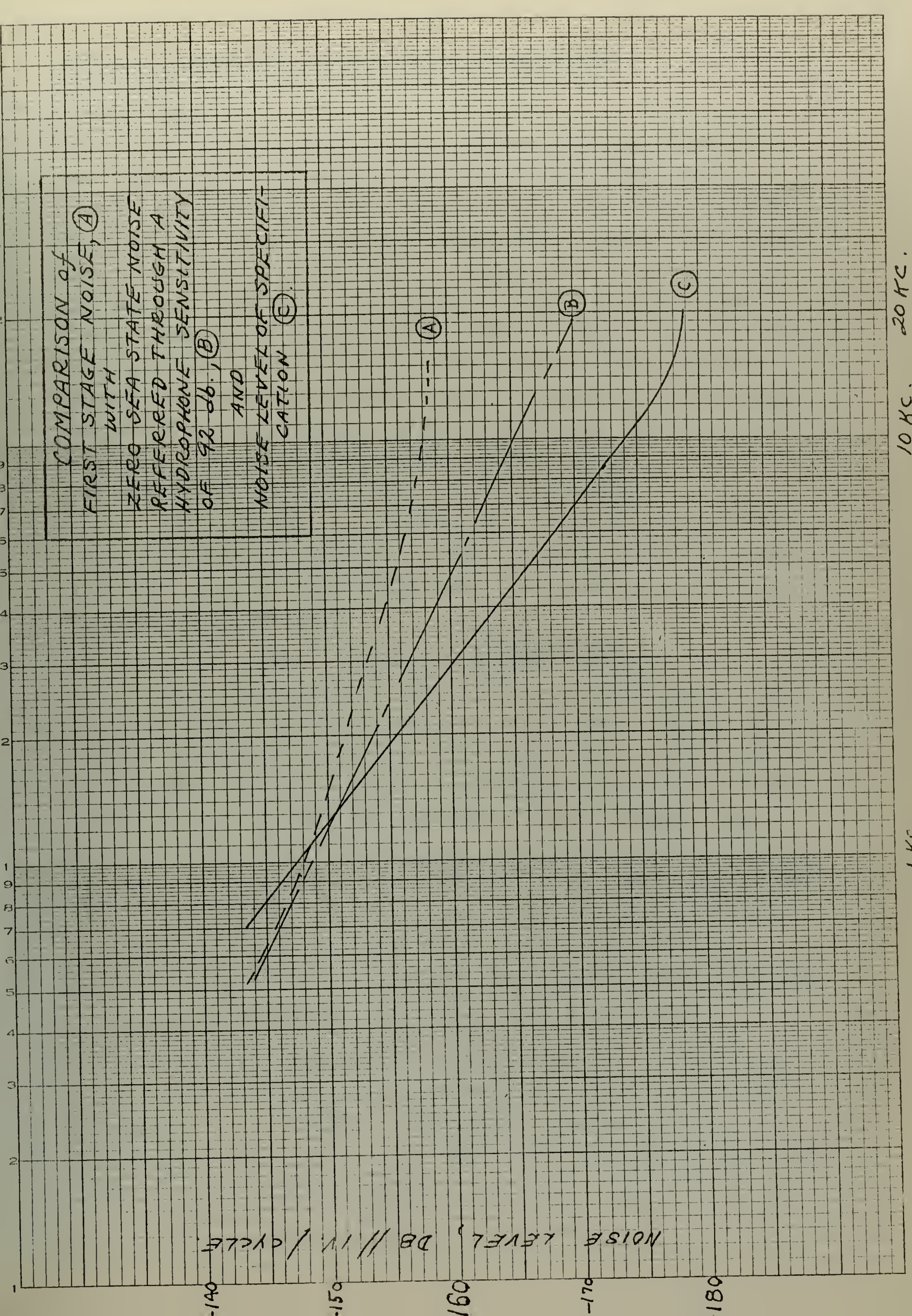
FIRST STAGE CIRCUITS
SUMMARY OF RESULTS
① BASIC TRIODE CIRCUIT (UN-BYPASSED R_K)
② CASCODE CIRCUIT
③ PARALLEL CIRCUIT
④ BEST TRANSISTOR
⑤ SELECTED CIRCUIT (SINGLE TUBE, BYPASSED R_K)
NOTE EXPANDED DB SCALE.
NOISE LEVELS, DB// 1 V./CYCLE
145
150
155
160
1 KC.
10 K
20 KC.
FREQUENCY

as pressure variations which are changed into electrical signals by the Sonar transducer, and applied to the pre-amplifier input. Thus the characteristics of the transducer, specifically its sensitivity and directivity, as well as sea noise itself, establish the minimum level at the pre-amplifier.

Sea noise is characterized by a -5db/octave slope.[22] The hydrophone sensitivity establishes its level in volts at the pre-amplifier input and the directivity of the hydrophone array changes the slope. For comparison with the curves on pg.29a. are shown two lines, one representing zero sea state referred through a hydrophone sensitivity of 92 db//1 u bar/cps, and one which represents the specific requirement for this pre-amplifier. This line includes the affects of both sensitivity and directivity of the particular hydrophone array to be used.

It is apparent that none of the first stage circuits investigated attains the required level at 20 KC. of -179 db//1 volt per cycle, nor was this to be expected from a survey of existing literature. It appears that the minimum level of a vacuum tube circuit in the present state of the art is approximately -160 db//1 volt per cycle in the 15 KC.to 20 KC. frequency range. In view of this, it would seem that for any requirement substantially lower than this figure, an input transformer becomes necessary.

Among other things, the foregoing investigation establishes that no undue penalties are involved in the use of sub-miniature tubes for low-noise applications of this sort. The fact that the first stage noise is considerably greater than required by the specifications in no way lessens the importance of designing for minimum noise of this stage.
As we shall see in the next section, limitations in the input transformer circuit further emphasize this importance.


COMPARISON of
FIRST STAGE NOISE, (A)
WITH
ZERO SEA STATE NOISE
REFERRED THROUGH A
HYDROPHONE SENSITIVITY
OF 92 db., (B)
AND
NOISE LEVEL OF SPECIFI-
CATION (C)
(A)
(B)
(C)
1 KC.
10 KC.
20 KC.
FREQUENCY
NOISE LEVEL DB // 1V / CYCLE
-140
-150
160
-170
180

STEP 3. SELECTION OF THE INPUT TRANSFORMER

Turns Ratio

Having established the minimum level of noise in the first stage of amplification, the next step is to select a transformer turns ratio which will supply a gain equal to or greater than the difference between this level and the required level. If our first tube stage level is -160 db and -180 db is required, we need a step-up transformer gain of 20 db. But this is more easily stated than accomplished. We can achieve the required gain easily enough, but the transformer itself contains noise sources. In addition, its inductance and capacitance have important effects. These, we will discuss later, limiting our present consideration to the problem of noise.

Theoretically, the noise of a transformer consists only of thermal agitation in its resistive component seen looking into the input terminals. The maximum value this quantity can have is:

$$R_{eq} = R_{primary} + \frac{R_{secondary}}{(\text{turns ratio})^2}$$

When one considers that the noise specification of -179 db//1 volt at 20 kc. represents a noise voltage of 1.11 X 10^{-9} volts on a per cycle basis, and corresponds to the thermal agitation noise in a 74-ohm resistance, it is seen that the primary and secondary winding resistance, and the turns ratio, are matters requiring careful thought in obtaining the required amount of "low noise" transformer gain. As a matter of fact, the requirement is even more difficult because the first stage noise must also be referred back through the transformer gain. This means that

$$R_{primary} + \frac{R_{secondary}}{n^2} + \frac{\text{1st stage } R_{eq}}{n^2} \leq 74 \text{ ohms.}$$

With n being the turns ratio.

Thus the importance of a minimum noise first stage is apparent. A level of -160 db//1v. corresponds to R_{eq} of 4090 ohms. For a transformer gain of 20 db, we need a turns ratio of 10, so $\frac{4090}{10^2} = 40.9$ ohms, leaving only 33 ohms for the transformer R_{eq}.

On this basis alone, n should be as large as practicable. Unfortunately, there are other effects which limit the turns ratio, and these effects are more difficult to analyze.

Other Effects Producing Noise in the Transformer

As the turns ratio is increased, one may expect more susceptability to hum and stray pickup. Presumably, close attention to isolation and shielding will help here, but in view of the levels involved, these measures may be rather extreme in extent. Another effect which may become important at low levels is BARKHAUSEN noise, which is given by the equation (19):

$$\overline{e^2} = 16\ \omega N^2 A^2 m B_{max} 10^{-16} \int_{f_1}^{f_2} df \qquad \text{(Eqn. 6)}$$

where $\overline{e^2}$ = Mean square output voltage

N = number of turns on output coil

A = cross section of core

m = magnetic moment of elementary magnet

V = volume of the core

ω = angular frequency of periodic magnetizing force.

B_{max} = maximum flux density

Barkhausen noise is said to be produced by the action of the elementary magnets within the core material. They orient themselves with the direction of the increasing field, but do so discontinuously. Although Eqn. 6. contains ω of a periodic magnetizing force, it would seem that the source of noise would exist whether the magnetizing force were periodic or random.

The possibility that hum or stray pickup voltages (e.g.,60-cycle) can induce Barkhausen noise should not be overlooked. Sixty cycle pickup itself lies outside the useful frequency band and is therefore no problem. If, however, it constitutes the ω in Eqn. 6, we have a broad-band noise source. This points to the requirement of carefully shielding a low-level transformer from pickup of any spurious voltage, regardless of frequency.

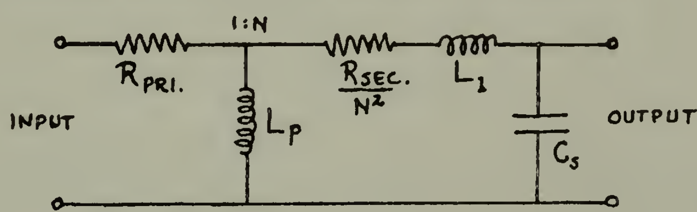
In this investigation, the transformer shielding was less than perfect. In any case, calculation of the magnitude of Barkhausen noise for the case at hand is beyond the scope of this paper. Nor can it be certified that this effect actually did exist. The significance of N should be noted, however.

Input Transformer - Results

The specification and attainment of all the desired parameters of a transformer is difficult, especially in sub-miniature sizes. The transformer used by the authors had the following specified characteristics:

Turns ratio = 20
Primary D.C. Resistance = 37 ohms
Secondary D.C. Rèsistance = 6700 ohms

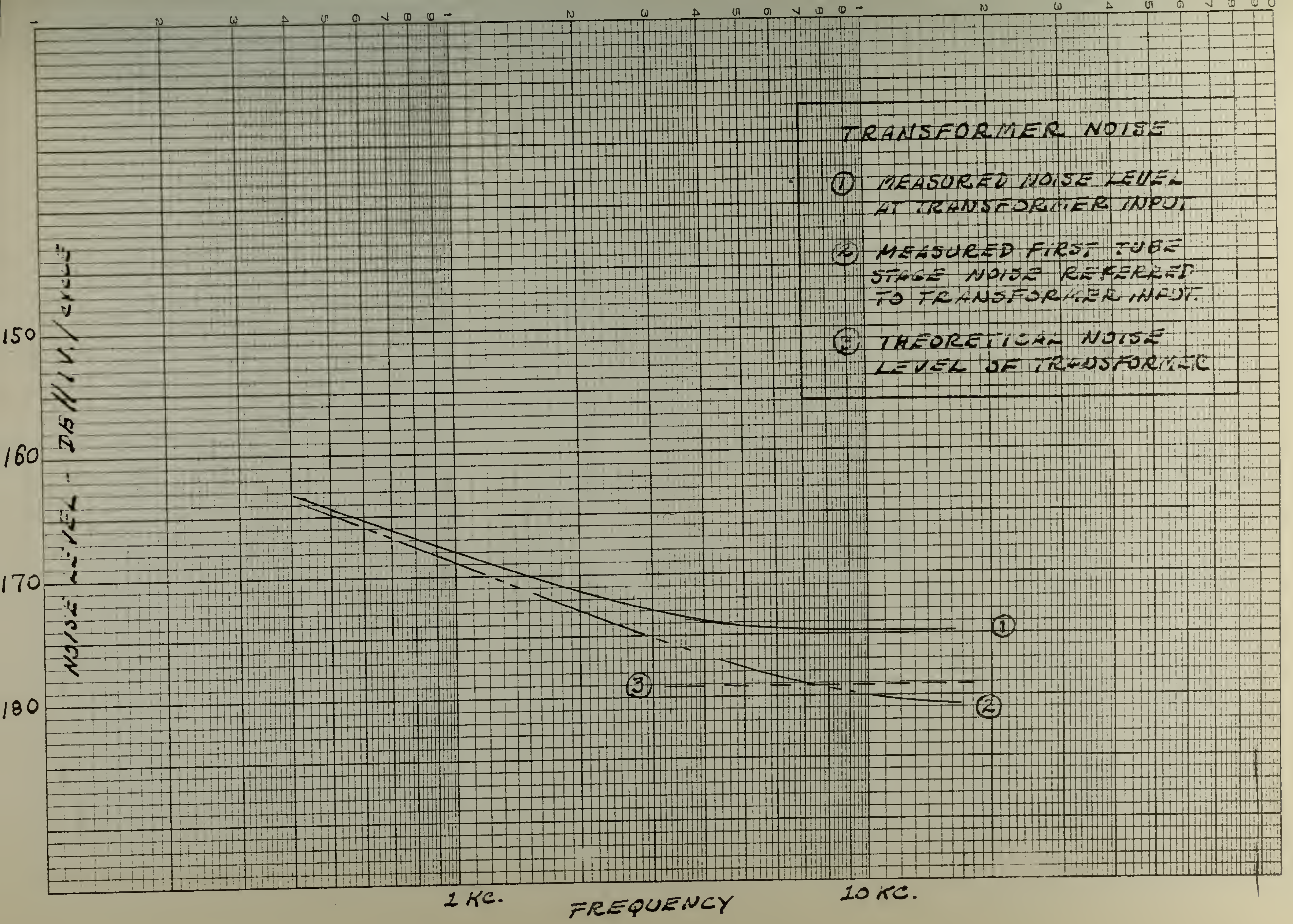
Its equivalent circuit is given in Fig. 10



where L_p = primary inductance
L_l = leakage inductance referred to the primary
C_s = stray and distributed capacity referred to the primary.

Fig. 10. Transformer Equivalent Circuit.

The measured noise level of this transformer is shown on page 32a, together with the measured noise level of the first stage referred to the transformer primary. The latter is at least 3 db below the transformer noise curve. The dotted curve on page 32a gives the theoretical noise level of the transformer based on thermal agitation due to its resistive component (57 ohms). The discrepancy between the theoretical and measured levels is attributed to Barkhausen effect induced by 60-cycle pickup.


TRANSFORMER NOISE
1 MEASURED NOISE LEVEL AT TRANSFORMER INPUT
2 MEASURED FIRST TUBE STAGE NOISE REFERRED TO TRANSFORMER INPUT.
3 THEORETICAL NOISE LEVEL OF TRANSFORMER
1
2
3
150
160
170
180
NOISE LEVEL - DB//V/CYCLE
1 KC.
10 KC.
FREQUENCY

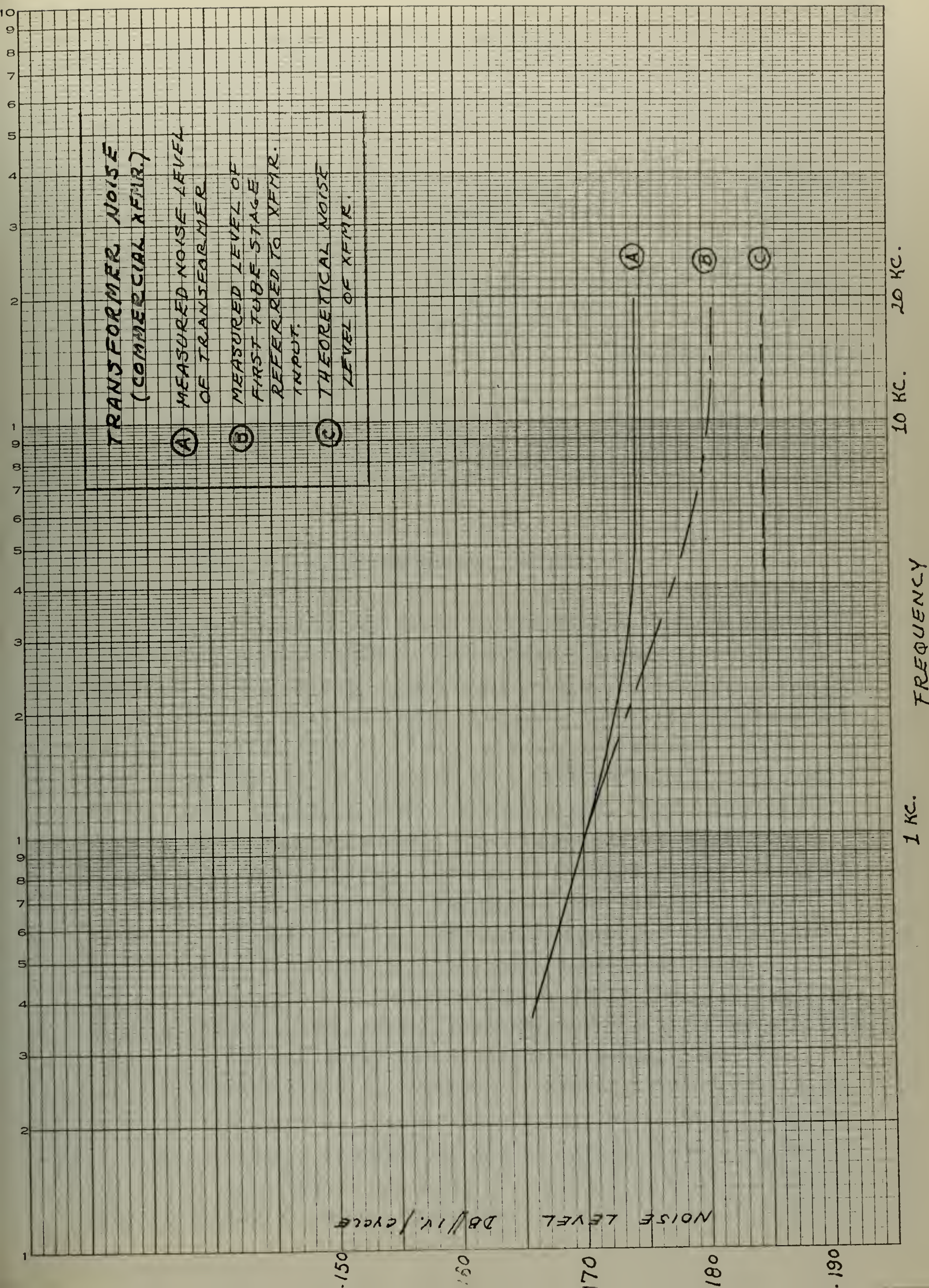
The sloping rise of the total noise curve at the lower end of the band indicates the effect of flicker noise which becomes greater than transformer noise at about 1Kc, even though the flicker level is reduced by the turns ratio gain of the transformer.

The stray and distributed capacitance of the windings of a transformer can also set a limit to the turns ratio available in a transformer. Once the value of this capacitive reactance, referred to the primary, becomes comparable to the source impedance, the turns ratio gain cannot be realized by the transformer. This will limit the amount by which the first stage noise is reduced.

The measurements shown of page 32a suggest that the transformer noise sets the ultimate limit in reduction of pre-amplifier noise. To help confirm this analysis, a subminiature commercial (TRI-AD) transformer was tested. It had the following parameters:

Turns Ratio	=	20:1
Primary Resistance	=	7 Ω
Secondary Resistance	=	3700 Ω

Its equivalent resistance was therefore equal to only 16 ohms and its measured and theoretical noise levels are also shown, on page 33 a., thus confirming the existance of some effect not ascribable to thermal agitation alone.


TRANSFORMER NOISE
(COMMERCIAL XFMR.)
(A) MEASURED NOISE LEVEL OF TRANSFORMER
(B) MEASURED LEVEL OF FIRST TUBE STAGE REFERRED TO XFMR. INPUT.
(C) THEORETICAL NOISE LEVEL OF XFMR.
A
B
C
1 KC.
10 KC.
20 KC.
FREQUENCY
NOISE LEVEL DB//V./CYCLE
-150
160
170
180
-190

THE PRE-AMPLIFIER CIRCUIT

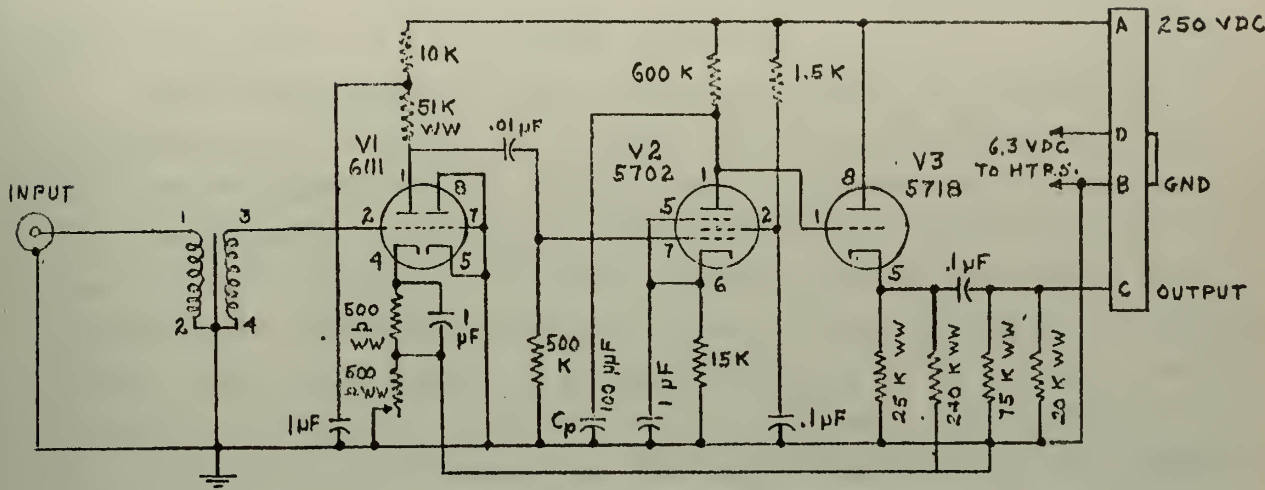
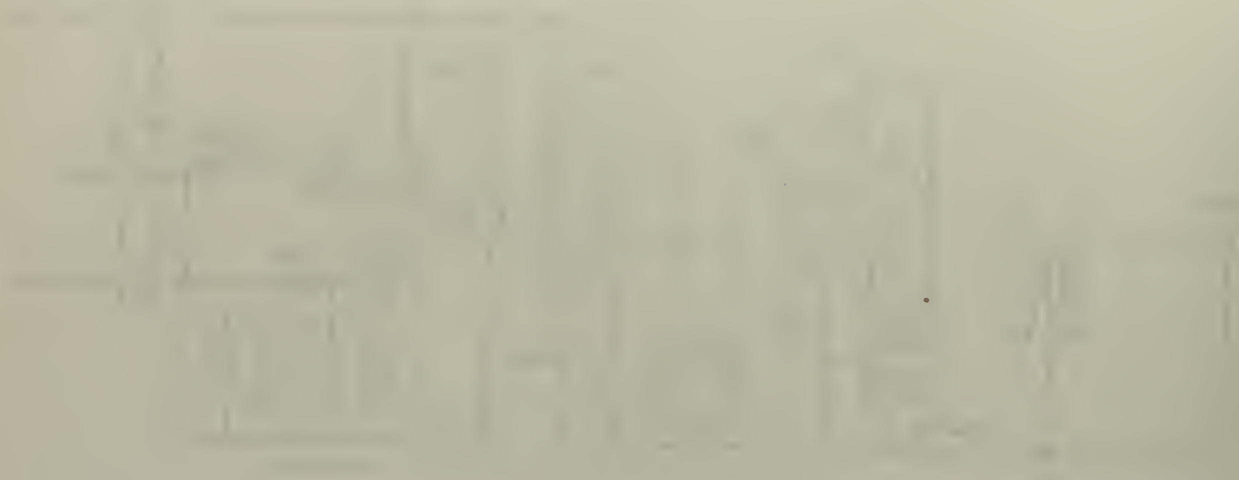


Fig. 11 The Pre-Amplifier Circuit.

Fig. 11 presents the final pre-amplifier circuit derived principally on the basis of low noise. It is a conventional feedback amplifier with a mid-band gain of 70 db. As a result of the study of various input circuits, the first stage is a triode amplifier with by-passed cathode resistance, and a gain of 22 db. It is followed by a pentode having a gain of 38 db, and a cathode follower output stage. The transformer gain is 26, giving an overall open loop gain of 86 db and allowing 16 db of inverse feedback to provide gain stability.

The overall frequency response of the amplifier is as shown on pg.35a.(curve A). The low frequency peak is caused by the *series* resonance of the source capacitance and primary inductance of the transformer, while the high frequency peak is caused by the series resonance of the transformer leakage inductance and the stray and distributed capacitance of the transformer circuit. Methods of shaping the overall response is discussed in greater detail in the following chapter.

The frequency response of the amplifier proper, without the input transformer, as shown on pg.35a.(curve B), is flat from below 100 cycles to approximately 60 kc. Direct coupling



between the pentode stage and the cathode follower reduces phase shift and attenuation at low frequencies. The purpose of C_p, from the plate of the pentode to ground, increases the attenuation at high frequencies in order to prevent oscillation.

The output impedance of the amplifier was less than 200 ohms over the frequency range, due to the cathode follower and the effect of the inverse feedback.

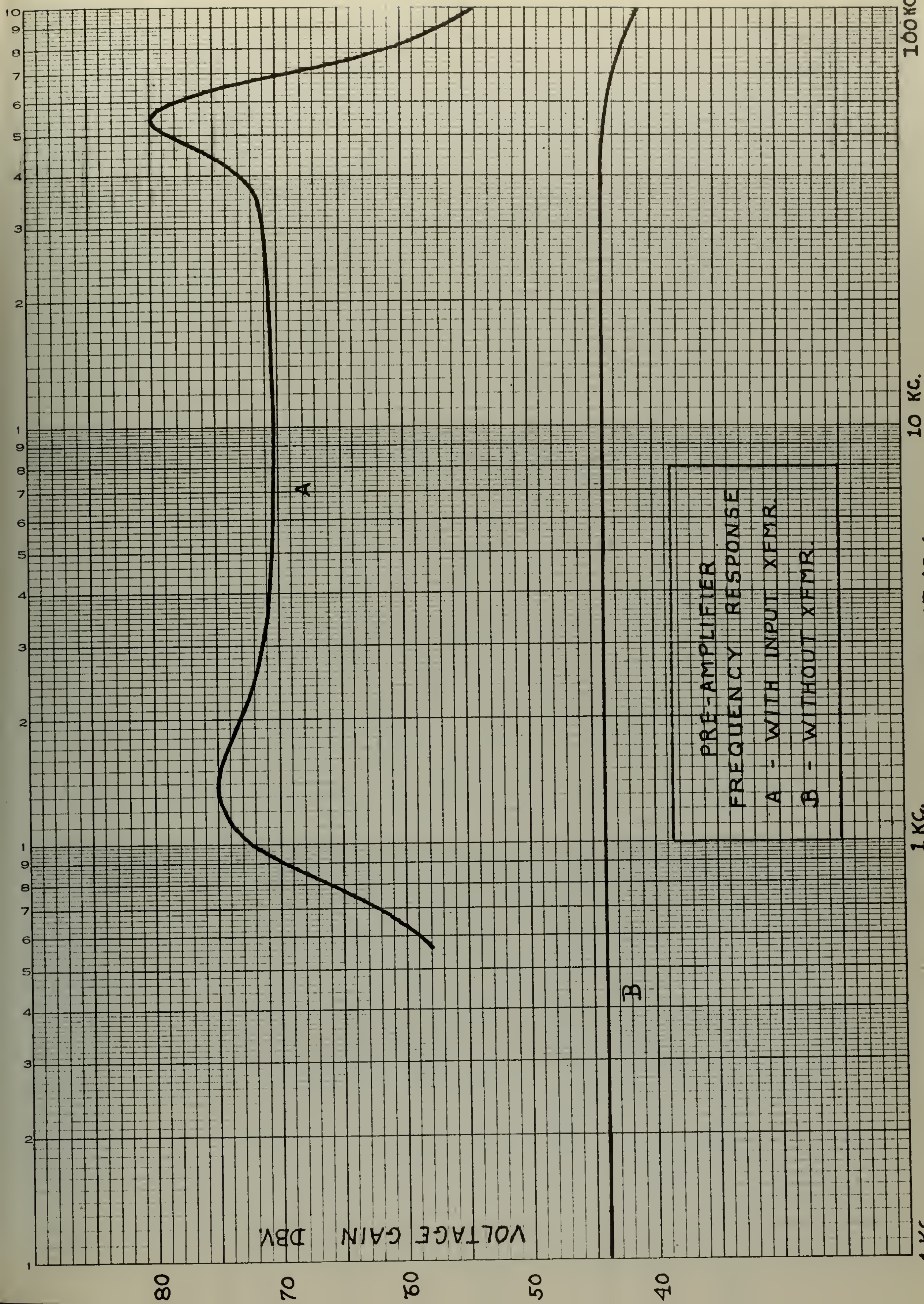
The performance of the amplifier proper is considered satisfactory, although it is felt that a greater amount of inverse feedback would be desirable, particularly in view of the requirement of matching 48 identical pre-amplifiers in each Sonar equipment. To obtain substantially more gain (and hence, more feedback), another stage of amplification is required. While this would occasion a small increase in size, the greatly improved stability characteristics would more than justify it.

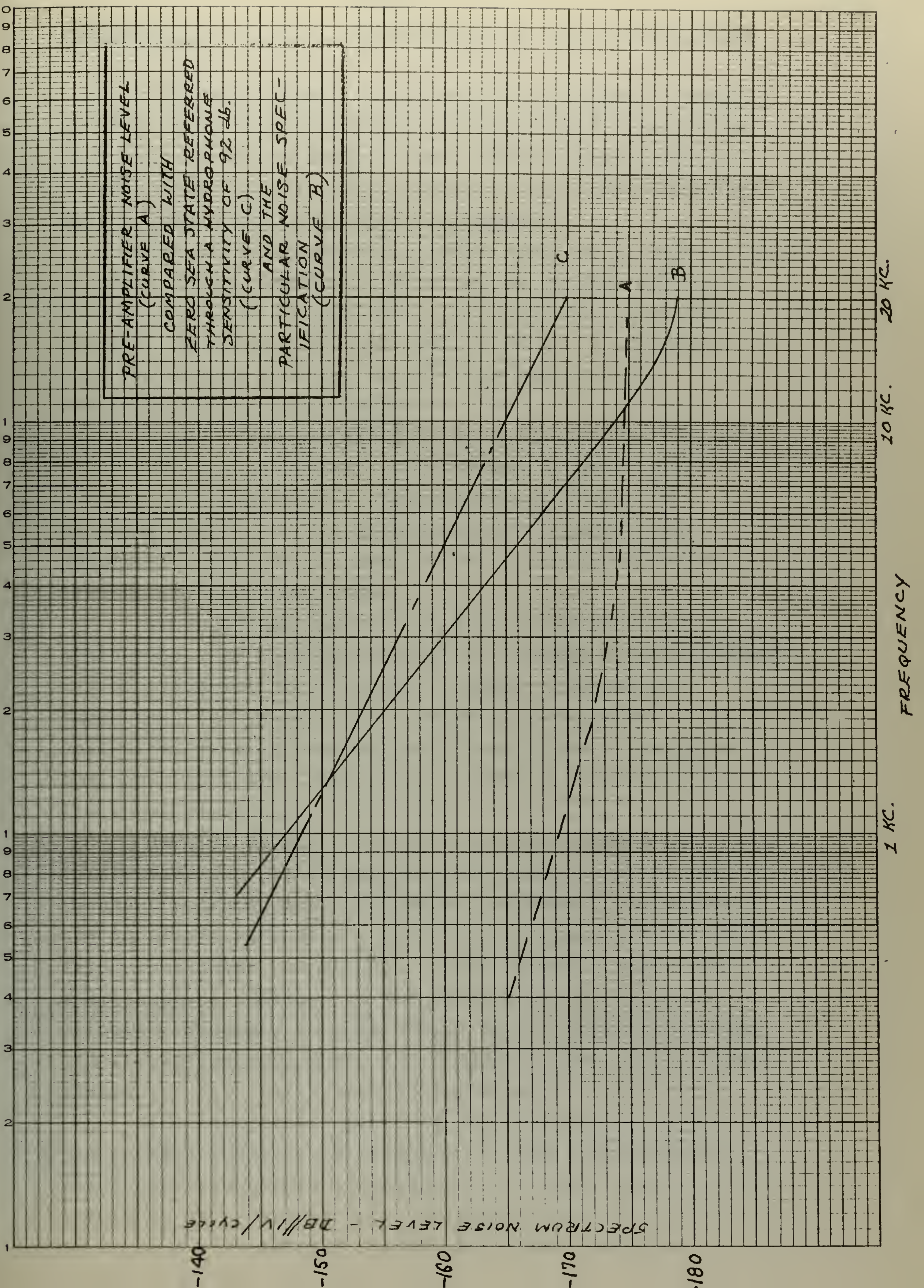
As previously indicated, the major emphasis of the design was on the noise level. The measured noise level of the final amplifier design is shown on pg 35b, where it is compared with the requirements of the specification. A curve of minimum sea noise, referred through the transducer, but assuming the transducer non-directional, is also presented for comparison. As can be seen from the figure, the measured noise level intersects the required noise level at approximately 11 kc, and exceeds the required noise level by 4 db at the upper limit of 20 kc.

The preamplifier noise, rather than sea noise, thus determines the magnitude of the minimum detectable signal in the upper part of the frequency range. Recommendations as to methods of further reduction of the preamplifier noise level are given in the final chapter.

A comparison of the preamplifier noise level with the level of zero sea state, referred through a non-directional transducer, (curve C on pg. 35b), shows that, in this case, sea noise, rather than amplifier noise, determines the minimum detectable signal strength.

This low noise level of the amplifier was achieved with little sacrifice of the miniaturization concepts. The test amplifier, as constructed, occupied a volume of 6 cubic inches. Since the test amplifier contained many capacitors and resistors of normal size, the authors estimate that the volume of the amplifier may be reduced even further by use of sub-miniature components.


PRE-AMPLIFIER
FREQUENCY RESPONSE
A - WITH INPUT XFMR.
B - WITHOUT XFMR.
VOLTAGE GAIN DBV
FREQUENCY
.1 KC.
1 KC.
10 KC.
100 KC.
80
70
60
50
40
A
B


PRE-AMPLIFIER NOISE LEVEL
(CURVE A)
COMPARED WITH
ZERO SEA STATE REFERRED
THROUGH A HYDROPHONE
SENSITIVITY OF 92 db.
(CURVE C)
AND THE
PARTICULAR NOISE SPEC-
IFICATION
(CURVE B)
A
B
C
1 KC.
10 KC.
20 KC.
FREQUENCY
SPECTRUM NOISE LEVEL - DB//1V/cycle
-140
-150
-160
-170
-180

STEP 4

CONSIDERATION OF OTHER REQUIREMENTS OF THE PRE-AMPLIFIER

Although minimum noise level is a primary consideration, other requirements may prevent design decisions from being based on noise alone. Chief among these are source impedance, frequency response, and gain stability.

Source Impedance and Frequency Response

The SONAR Hydrophone in this case has an essentially reactive source impedance which may be represented by a series capacitor of .025 ufd. The specified frequency response requires a voltage gain of 70 db. flat within ± 0.5 db between 1 kc and 20 kc. The asymptotic response is shown on page 39a.

In the previous section, some of the restrictions on transformer parameters with respect to the noise problem were explained. The additional factors of a reactive source and frequency response impose other important restrictions, most of which are inconsistent with the noise requirement.

In an attempt to resolve some of these inconsistencies, and keeping in mind that future specifications may be different as to noise, response, and source impedance, the writers have sought to develop an analytical means of correlating these three factors in the case where the frequency response is determined in the input circuit.

Control of the frequency response in this way consists of the use of damping resistors to eliminate resonant peaks in the transformer response. The method has important advantages, but from what has already been said about noise, we should, at the outset, regard with suspicion the addition of any resistance in the input circuit. However, in some cases, this may be permissable and for the general case, the analysis given may indicate whether or not this system is feasible.

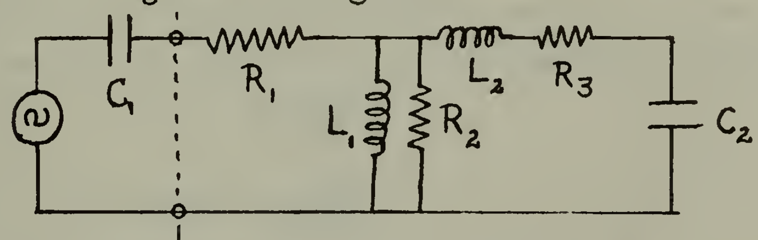
The analysis is given in the Appendix, page 57. The results involve the solution of four simultaneous non-linear equations, the solution of which is so laborious as to render them limited in practical value. In a great many cases, however, the upper and lower

break frequencies are sufficiently separated so that the problem can be divided into two parts, and a usable solution obtained with the aid of a few simplifying assumptions. The derivation of this special case is given in the Appendix, page 62., and the results are given in the following section.

Special Case

The problem is to determine the required transformer primary inductance, shunt capacitance, and series and shunt damping resistances to shape a given frequency response, and from these to calculate the resultant noise level at any frequency.

The transformer equivalent circuit used in the derivation is given in Fig. 12.



R_1 = series damping R + primary R

R_2 = shunt damping R, referred to primary

R_3 = Secondary R, referred to primary (assumed small compared to R_1 and R_2)

L_1 = Primary inductance

L_2 = Leakage inductance

C_1 = Source impedance (given)

C_2 = Distributed and added shunt capacitance

Fig. 12.

For a flat response of the type shown in Fig. 13, the upper and lower break frequencies, ω_1 and ω_2, are given:

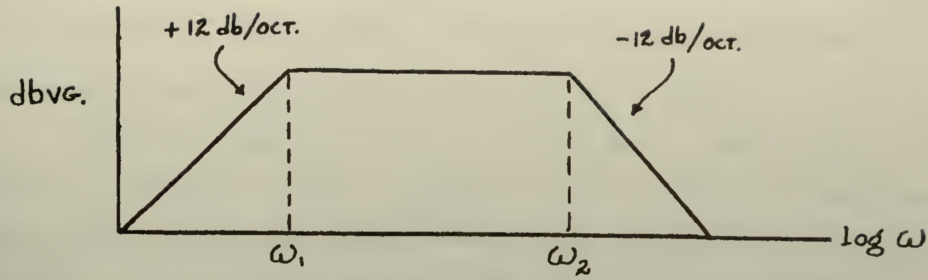


Fig. 13

With the following assumptions:

$$\omega_2 \gg \omega_1$$

$$R_2 \gg R_1$$

$$C_1 \gg C_2$$

the derivation (Appendix, pg.62) results in the following relationships to establish the frequency response:

$$L_1 = \frac{1}{\omega_1^2 \; C_1^2}$$

$$R_2 = \frac{1}{2 \; \omega_1 C_1}$$

$$R_1 = 2\omega_2 L_2$$

$$C_2 = \frac{1}{\omega_2^2 L_2}$$

As a result of the work shown on pg.58 of the Appendix, the mean square noise voltage of the transformer circuit may now be predicted by the following equations:

For low frequencies:

$$\overline{e}_n^2 = 4KT \left[R_1 + \frac{R_2 L_1^2 \omega^2}{\left[R_2 \; (1-L_1C_2\omega^2)^2 + L_1^2\omega^2 \right]} \right] \qquad \text{(Eqn.1)}$$

For high frequencies:

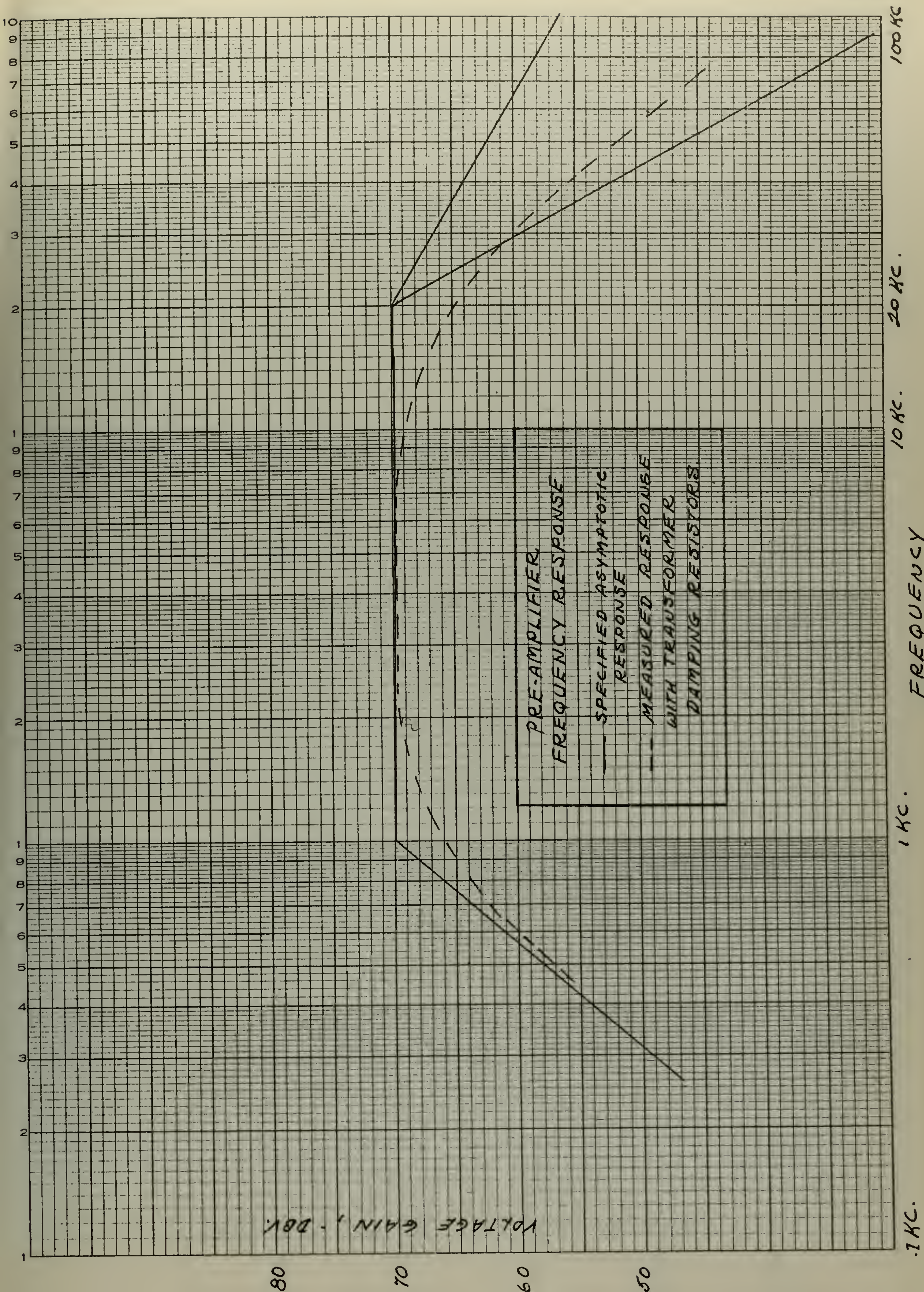
$$\overline{e}_n^2 = 4KT \left[R_1 + \frac{R_2}{1 + \frac{R_2^2 C_2^2 \omega^2}{[1-L_2C_2\omega^2]^2}} \right] \qquad \text{(Eqn.2)}$$

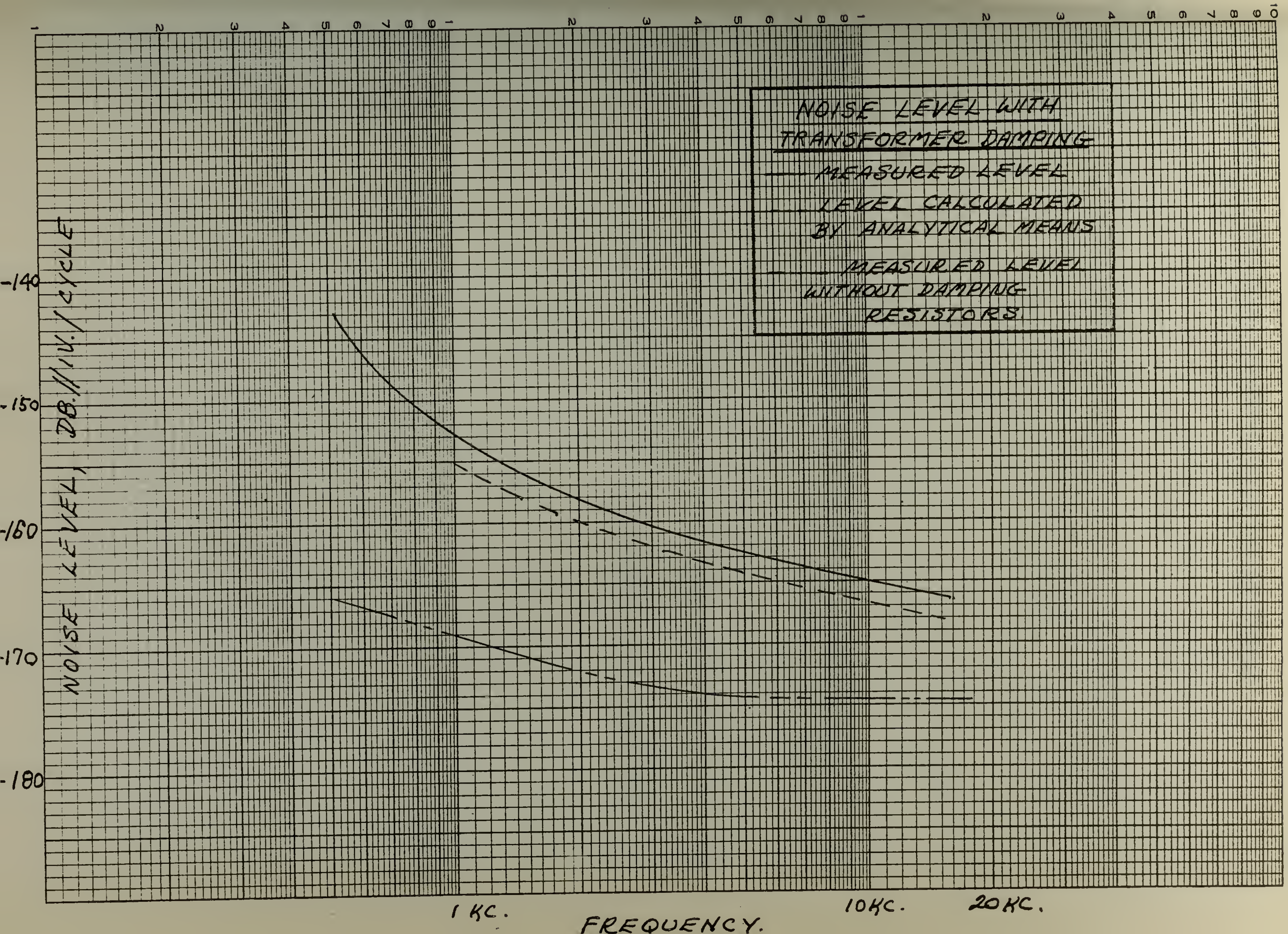
where K is Boltzmann's constant = 1.37×10^{-23} joules / $^\circ$K

T = Kelvin temperature

These expressions show the effect of R_1 at all frequencies, The effect of R_2 on noise depends on frequency as well as other circuit values.

This procedure was applied to a particular problem and the results agree closely with measured results. These are shown on page 39b. For comparison, is also shown a noise curve taken with no added damping resistance in the transformer circuit. The high cost of resistance in terms of noise is apparent.


PRE-AMPLIFIER
FREQUENCY RESPONSE
SPECIFIED ASYMPTOTIC RESPONSE
MEASURED RESPONSE WITH TRANSFORMER DAMPING RESISTORS
FREQUENCY
.1 KC.
1 KC.
10 KC.
20 KC.
100 KC.
VOLTAGE GAIN - DBV
80
70
60
50


NOISE LEVEL WITH
TRANSFORMER DAMPING
MEASURED LEVEL
LEVEL CALCULATED
BY ANALYTICAL MEANS
MEASURED LEVEL
WITHOUT DAMPING
RESISTORS
-140
-150
-160
-170
-180
NOISE LEVEL, DB.//1V./CYCLE
1 KC.
10KC.
20KC.
FREQUENCY.

Shaping of Frequency Response With Output Filter

There are many conventional ways to control the frequency response in an amplifier. For example, one simple way is to use an output filter as described in the following paragraphs.

In the preceding section, we noted the deleterious effects on noise of damping resistors in the transformer circuit. In particular, as indicated by Eqns. (1) and (2) page 38, R_1, the series damping resistor is the principle noise contributor at all frequencies. This resistor controls the high frequency response by critically damping the resonant peak due to the leakage inductance and the secondary shunt capacitance.

The series damping resistor may be eliminated by the use of an output filter to cancel the high end resonant peak and give the required roll-off. It is important, however, that the filter parameters be such as not to prejudice the miniaturization requirement. The analysis of the filter, based on complex frequency plane theory, is given in the Appendix pg. 65.

The results are shown on page 69 a. of the Appendix.

Chapter IV

Conclusions and Recommendations

Although the sonar preamplifier described in this report does not meet the required noise specification*, it shows a measured reduction in noise level of 7.5 db//1v per cycle when compared with the existing, full-sized equipment. The authors feel that this significant improvement justifies the design procedure which has been proposed. A summary of this design procedure, together with conclusions and recommendations which may aid in its application is presented in the following section.

Input Tube Selection: Based on the measured results, the noise level of circuits employing subminiature tubes closely approaches that of normal size low noise triode circuits in the frequency range from 1 Kc to 20 Kc. The transconductance of the tube presents a possible criterion for selection of tube type in the upper portion of the frequency range where shot noise is predominant. In applications where flicker noise is the major component, the work of van der Ziel[4] may serve as a guide. However, due to the wide variation in the flicker noise levels of tubes of the same type, as well as different types, measurement of these levels is recommended as the basis for tube selection.

The results of the noise level measurements of the transistor circuits indicate, under certain conditions, that they may closely approach the level of vacuum tube circuits. These results provide an excellent example of the recent progress in the transistor field, and may point the way to their employment in very low noise circuits.

Design of the First Stage of Amplification: Major emphasis should be placed on the reduction of noise in the first stage of amplification when applying this proposed procedure. The reason for this emphasis is that the noise level of the first stage determines the necessary turns ratio gain of the transformer, which is a major factor in determining the minimum noise level which may be obtained.

Of the various vacuum tube circuits investigated in this phase, the use of two triodes in parallel was found to have

* The specification referred to was issued in 1954, and the equipment is presently in the early stages of development.

the lowest noise level. The decision as to whether the reduction in noise level is worth the cost will depend upon the particular application.

The cascode circuit was investigated analytically and experimentally. Based on these results, the authors conclude that the cascode circuit does not present any appreciable improvement over the noise level of a high gain triode stage in the frequency range from 1 Kc. to 20 Kc. The study of the effects on noise level of introducing resistance into the grid to cathode circuit of the first stage, either for purposes of local degeneration, or overall feedback, indicate that such resistance should be kept to a minimum.

<u>Input Transformer Selection:</u> In the application of this design procedure, the minimum noise level which may be obtained is determined primarily by the transformer noise characteristics. The resistive component of the transformer windings, referred to the primary, places a lower limit on the level of theoretical thermal agitation noise which may be achieved. This resistive component will increase with an increase in turns ratio, for a given set of conditions. Since the turns ratio is determined by the amount which the first stage noise level has to be reduced, we can see the importance of the first stage noise level in determining the ultimate level of transformer noise. It should be pointed out that when the resistive component of the transformer windings, referred to the primary, becomes greater than the equivalent noise resistance of the first stage, also referred to the primary, further reduction in the total circuit noise cannot be achieved by increase in transformer turns ratio.

In this particular investigation, the ultimate limit in the circuit noise level was determined by effects other than the thermal agitation noise of the resistive components of the transformer.

Barkhausen effect is presented as a possible cause of this excess noise. In view of the importance of this excess transformer noise in determining the ultimate limit which may be obtained, the authors recommend that a thorough study of its causes and possible remedies be undertaken.

<u>Consideration of Other Requirements of the Amplifier:</u> A general rule concerning the design to provide characteristics other than low-noise, such as frequency response, is that the assignment to the input stage of any function which will increase the real part of its impedance will result in an increase in noise level.

APPENDIX

Method of Noise Measurement

The noise specification for the pre-amplifier as a whole is given in terms of decibels with respect to 1 volt on a per cycle basis (db//1v/percycle), as a function of frequency. Therefore the somewhat more familiar "noise figure" is not applicable.

The method employed is to make direct measurement at the output, with the input shorted. A G.R. 736-A Wave Analyzer is used as the output meter, and its reading for each discrete frequency, is referred back to the input through the total gain at that frequency and its own bandwidth correction. The 736-A has a bandwidth of 4-cycles, and thus readily lends itself to the measurement of rms. noise voltage on a per cycle basis. It has the further, and vastly more important, advantage that this narrow band excludes all extraneous signals. In particular, it completely eliminates 60-cycle pick-up voltage from the output.

Since r.m.s. noise voltage is proportional to the square root of the bandwidth, the values obtained with a 4-cycle band are very small. This requires a high gain (110-130 db) circuit for the measurement.

The equipment setup for the noise measurement is shown in the Appen. on pg. 45.

Noise Measurement Equipment and Test Arrangement

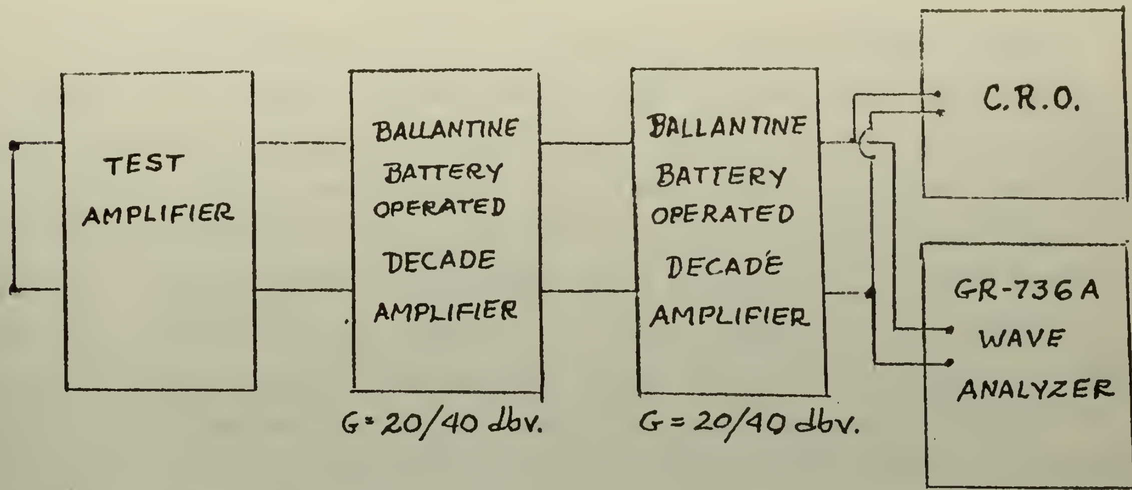


Fig 14. Test Equipment

The test arrangement for the noise measurement was as shown in the above figure. Noise measurements were conducted in the shielded room of the Electrical Communications Laboratory at M.I.T.

The input transformer, the pre-amplifier, and the battery power supply for the pre-amplifier, were encased within a shielding box during the measurements. All connection leads and plugs were shielded.

During measurements, the output noise voltage was monitored both visually, on the CRO, and aurally, on earphones at the output, to ensure that thermal noise, rather than non-random pickup, was being measured.

General Discussion of Low Noise Measurements

The principle difficulty in making noise measurements at very low levels is that great care is required to assure that one is measuring circuit noise alone and not some pick-up signal. The wave analyzer requires 60-cycle power and the writers experienced great difficulty with pickup from this source. The 60-cycle component does not appear at the output but, unless it is reduced to a low level, it will saturate the last decade amplifier, whose input is limited to 50 mv. When this happens all noise readings over the band are in error. Careful attention to shielding of all cables and equipment is mandatory. A.C. equipment not needed should be turned off. Battery supplies and equipment should be connected to a common ground, and it will be found that orientation of leads and equipment is important. Large capacitors should be connected across all battery terminals to by-pass battery noise to ground.

The output signal should frequently be checked on the oscilloscope for 60-cycle signal level or evidence of overloading, as well as aurally, with headphones, for its characteristic "hiss" and absence of any tone, or other non-random signal. Another effective means of checking for overloading is to reduce the gain of the next-to-last decade amplifier (see Fig. on pg. 45.). The output noise reading should drop an amount corresponding to 20 db. If it does not, the last decade amplifier has been overloading. The Ballantine decade amplifier has a 20-40 db gain switch so this check is quickly made.

Because of the many unforseen factors which can, without warning, give rise to inaccuracy in low-level noise measurements, it is important to confirm any noise stectrum data obtained, preferably at some other time. Any single set of readings cannot ever be considered conclusive. In many laboratories, there are a great many extraneous signals whose source and identity cannot readily be determined but which can, nevertheless, invalidate the noise measurements. Since even the presence of these spurious signals may not be suspected, the investigator in this field does well to be extremely careful.

The accuracy of the measurement equipment can be easily checked by connecting a calibrated variable resistor across

the input terminals. A noise measurement is made, and the value of resistance is then increased until the noise power output doubles (3db increase in $(\bar{e}_n)^2$ at the output). The value of the added resistance should equal the equivalent noise resistance of the circuit referred to its input.

Calculation of Noise Level at the Input on a Per Cycle Basis from the Measured Data.

A.) The noise voltages which were measured were the output noise voltages in the pass band of the GR 736-A Wave Analyzer. In order to refer these voltages to the input circuit, they must be divided by the gain of the stages between input and output. In order to determine the voltage for a one cycle band, a bandwidth correction factor must be applied to correct for the effect of the bandwidth of the measured noise. This bandwidth correction depends on the equivalent bandwidth of the measured noise.

Equivalent bandwidth is defined as follows:

$$B_{eq} = \frac{\int_0^\infty G_v^2(f)\,df}{G_v^2\ (\text{midband})}$$

Where G_v = Voltage Gain

Since the noise voltage is proportional to the square root of the bandwidth, the bandwidth correction factor is equal to $\sqrt{B_{eq}}$.

For the GR-736-A, the measured value of B_{eq} equals 4.16 cps.

$$\sqrt{B_{eq}} = 2.05$$

B.W. Corr. Factor = - 6.26 db//$_{1v}$

B.) Sample Calculation:

Measured (e_{noise}) at output = 4.0 mv = -48.2 db//1v.

Amplifier Gain = -108.3 db//1v.

B.W. Corr. Factor = - 6.3 db//1v.

(e_{noise}) at input = -162.8 db//1v/per cycle

It is , of course, very necessary that the amplifier gain over the whole band be accurately known. This requires a separate response curve for every change in circuit configuration or parameter values, as well as frequent checks on the gain of the decade amplifiers.

THEORETICAL NOISE LEVEL OF THE AMPLIFIER

A.) Representation of Shot Noise in a Triode at Low Frequencies

In the frequency range where shot noise is the major component of tube noise, the equivalent circuit of Fig. 15 . represents the noise generating properties of a triode. The current source, i_n, represents the noise current which would flow in the plate circuit if the plate were short-circuted to the cathode.

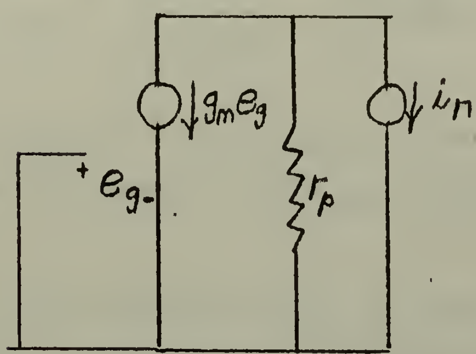


Fig. 15. Noise Equivalent current source.

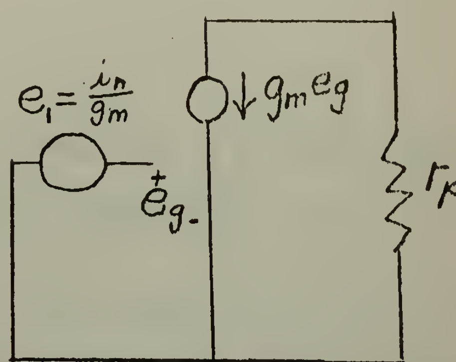


Fig. 16. Noise Equivalent voltage Source.

It is described by the following equation:[3]

$i_n = \sqrt{\Gamma^2 2eIp\,\Delta f}$

e = electron charge in coulombs (1.59×10^{-19})

Γ = a constant depending on the randomness of an electron

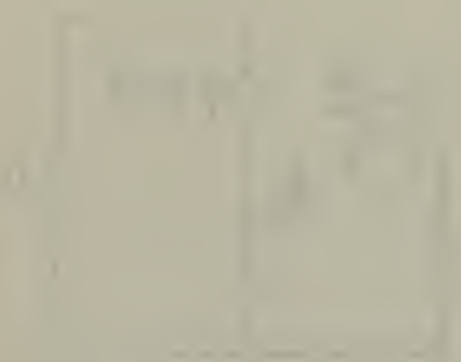
Ip = average plate current

Δf = bandwidth

i_n = root mean square value of noise current components.

A more convenient representation is shown in Fig. 16 , in which the noise current generator in the plate is replaced by an equivalent voltage generator in the grid circuit, e_1. This voltage generator, $e_1 = i_n/g_m$, is the root-mean-square value of noise voltage in the grid circuit which will produce the same effect as the current generator, i_n, in the plate circuit.

In order that the relationships between shot noise and the other noise components of a circuit may be visualized more readily, it is convenient to express e_1 in terms of an effective



resistance which would produce an equivalent noise voltage in accordance with the equation:

$$e_1 = \sqrt{4KTB\,R_{eff}}$$

The expression for R_{eff} may be derived as follows:

$$e_1^2 = 4KTBR_{eff} = \frac{i_n^2}{g_m^2} = \frac{\Gamma^2 2eI_p\Delta f}{g_m^2}$$

$$R_{eff} = \frac{\Gamma^2 2eI_p\Delta f}{g_m^2 4KTB} = \frac{\Gamma^2 eI_p\Delta f}{g_m^2 2KTB}$$

North,[12] and others, have shown that, for a triode, the above expression for R_{eff} simplifies to: $R_{eff} = \frac{2.5}{g_m}$

The representation of tube noise as an effective resistance greatly facilitates calculation of the theoretical noise level of a circuit. Since statistically independent noise voltage components add quadratically, the effective noise resistance of the tube, which is proportional to e_1^2, may be added directly to other resistive noise components in the grid circuit. This method is applied in the calculation which follows.

The reader is forewarned the above development is valid only for application where the following restrictions apply:

1.) The frequency is low enough that induced grid noise is negligible.
2.) The interelectrode capacitances are small enough so that their effects may be neglected.
3.) Flicker effect noise has been neglected. The flicker noise may, however, also be expressed as an effective noise resistance in the grid circuit as may be shown by a similar derivation.

The first stage of the amplifier is shown in Fig. 17 .

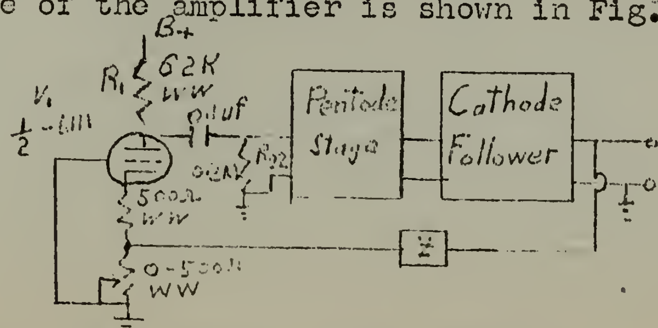


Fig. 17. Basic First Stage Circuit

The total circuit noise may be referred to the grid of the first tube in the form of an equivalent resistance (Req). The noise voltage may be calculated by the equation:

$$Erms = \sqrt{4\ KTBReq}$$

$K = 1.37 \times 10^{-23}$ (Boltzmann's constant)

$T = 300°$ Kelvin (assumed temperature of 70° F)

Beq = Bandwidth = 1c.p.s.

Req = Total equivalent noise resistance in the grid circuit.

The first step is to calculate the total equivalent noise resistance in the grid circuit. This noise resistance is primarily determined by the shot noise of the first stage tube, and the noise voltage in the plate and cathode resistance of the first stage.

Noise voltages from succeeding stages may be neglected since, by referring them to the first stage grid, they are attenuated by the voltage gain of the intermediate stages.

B.) <u>Calculation of Total Req.</u>

1.) Reduced Shot Effect Noise of First Stage Triode

$$R_{eff.} = \frac{2.5}{g_m} = \frac{2.5}{5000 \times 10^{-6}} = 500 \text{ ohms}$$

2.) Noise Voltage in the Plate Circuit

The noise components present in the plate circuit are R_1, R_{g2}, and the tube noise of the pentode in the second stage. The equivalent noise resistance present from grid to ground of the second stage is as follows:

$$(Req.)_{\text{grid of v2}} = (R_{eff.})_{\text{pentode}} + Ri \left[\frac{r_p}{r_p + Ri}\right]^2$$

$$\text{Where } Ri = \frac{R_L R_{g2}}{R_L R_{g2}} = \frac{62X10^3 X200X10^3}{262X10^3} = 47.3K$$

$$Ri \left[\frac{r_p}{r_p + R_i}\right]^2 = 47.3 \left[\frac{5K}{5K + 47.3K}\right]^2 = 0.438K$$

$$(R_{eff.})_{\text{pentode}} = 24K \text{ (measured)}$$

$$(Req)_{\text{grid of } V_2} = 24K + 0.438K \approx 24.4 \text{ K}$$

This value must be divided by the square of the first stage voltage gain (100), to refer it to the first stage grid.

$$(Req.) \text{ referred to grid of } V_1 = \frac{24.4K}{100} = 244 \text{ ohms}$$

3.) Noise Resistance in the cathode circuit

The noise resistance in the cathode circuit is the sum of the two cathode resistances.

$$R_{cath.} = 500 + 250 = 750 \text{ ohms.}$$

4.) Total Equivalent Noise Resistance

$$\text{Total Req.} = R_{shot} + R_{plate} + R_{cath.}$$

$$\text{Total Req.} = 500 + 244 + 750 = 1494 \text{ ohms.}$$

C.) <u>Comparison of Measured and Calculated Levels</u>

This value of total equivalent noise resistance does not include any allowance for flicker noise, due primarily to the lack of a satisfactory equation to describe this

noise for sub-miniature tube types. It is possible that flicker noise might be greater in the sub-miniature tube types than in normal size tubes due to the smaller cathode area. However, since the plots of the noise levels of all the circuits measured started to flatten out at frequencies between 5 Kc. and 10 Kc., it may be concluded that the flicker noise level is below the level of the sum of the other noise components at the high end of the frequency range. Based on this, an effective flicker noise resistance of 400 ohms is arbitrarily assigned.

The total equivalent noise resistance, referred to the input grid, is then 1894 ohms. This corresponds to a noise voltage:

$(e_n) = -\ 164.2$ db//1 volt per cycle — calculated

This value is compared with the measured value at 16 Kc. The highest measurement frequency is selected in order that the unpredictable effect of flicker noise may be minimized.

$(e_n) = -\ 156.8$ db//$_{1\text{volt}}$ per cycle — measured

The discrepancy of 7.4 db between the calculated and measured values of noise level appears rather large. Possible causes of this discrepancy are as follows:

1.) Measurement accuracy limitations due to the extremely low levels involved.
2.) Inexactness of theoretical value of flicker noise,
3.) Incomplete elimination of external noise sources, particularly battery noise, in the measurement circuit, e.g. battery operated decade amplifiers.

Noise Analysis of Cascode Circuit

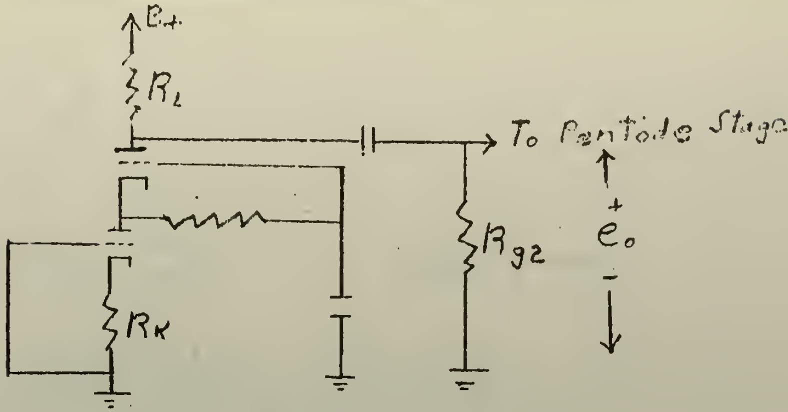


Fig. 18. Actual Test Circuit

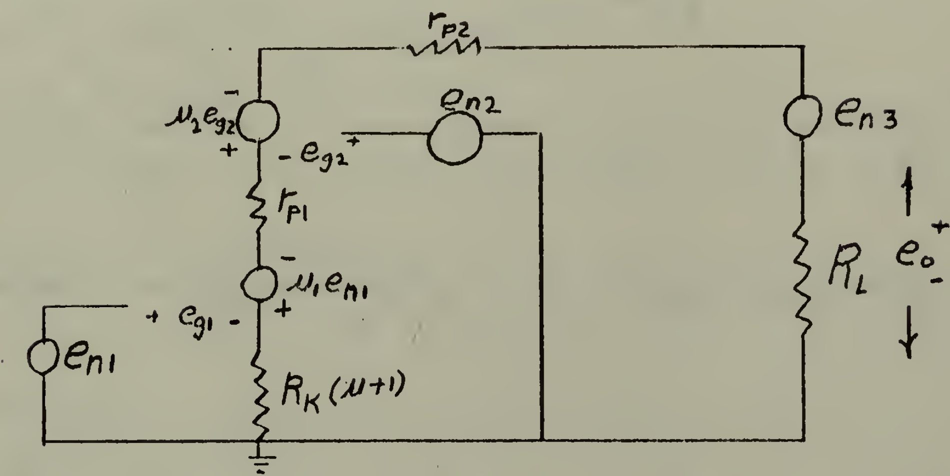


Fig. 19. Incremental Piecewise Linear Equivalent Circuit

The incremental piecewise linear equivalent circuit permits a simplified noise analysis of the cascode circuit in the frequency range of interest. The noise components present in the circuit may be defined as follows:

e_{n1}= Total noise voltage in the grid to cathode circuit of V_1, consisting of the tube noise of V_1 and the thermal agitation noise of R_k.

e_{n2}= Effective tube noise voltage of V_2.

e_{n3}= Noise voltage due to the tube noise of the succeeding stage and the thermal noise of R_L and R_{g2}.

The total equivalent noise level present at the grid of V_1 may be calculated as follows:

$$(e_n)_{\text{referred to input}} = \sqrt{e_{n1}^2 + e_{n2}^2 \left(\frac{G_2}{G_1}\right)^2 + \frac{e_{n3}^2}{G_1^2}}$$

where $G_1 = \dfrac{e_o}{e_{n1}}$

$$G_2 = \frac{e_o}{e_{n1}}$$

To find G_1:

$$G_1 = \frac{e_o}{e_1} = \frac{-ipR_L}{e_1} \qquad \textit{where } e_1 = e_{n1}$$

$$i_p = \frac{u_1 e_1 + u_2 e_{g2}}{R_L + r_{p2} + r_{p1} + (u+1)\, RK}$$

where $e_{g2} = u_1 e_1 - ip\left[r_p + R_k\,(u_1+1)\right]$

$$i_p = \frac{u_1 e_1 + u_2\left\{u_1 e_1 - i_p\left[r_p + R_k (u_1+1)\right]\right\}}{R_L + r_{p2} + r_{p1} + (u+1)\, R_k}$$

$$i_p = \frac{u_1 e_1 + u_1 u_2 e_1}{R_L + r_{p2} + r_{p1} + (u_1+1)\, R_k + u_2 r_{p1} + u_2\, R_k\,(u_1+1)}$$

$$G_1 = \frac{-i_p R_L}{e_1} = \frac{-u_1\,(u_2+1)\,R_L}{(u_2+1)\, r_{p1} + r_{p2} + R_L + (u_1+1)\,(u_2+1)\, R_K}$$

To find G_2:

$$G_2 = \frac{e_o}{e_{n2}} \quad \text{for } e_{n1} = o$$

$$G_2 = \frac{-i_p R_L}{e_{n2}}$$

$$i_p = \frac{u_2 e_{g2}}{r_{p1} + r_{p2} + (u_1+1)\, R_K + R_L}$$

$$i_p = \frac{u_2\left\{e_{n2} - i_p\left[r_{p1} + (u_1+1)\, R_K\right]\right\}}{r_{p1} + r_{p2} + (u_1+1)\, R_K + R_L}$$

$$i_p = \frac{u_2 e_{n2}}{r_{p1} + r_{p2} + (u_1+1)\, R_K + R_L + u_2\, r_{p1} + u_2\,(u_1+1)\, R_K}$$

$$i_p = \frac{u_2 e_{n2}}{(u_2+1)\, r_{p1} + r_{p2} + R_L + (u_2+1)\,(u_1+1)\, R_K}$$

$$G_2 = \frac{-i_p R_L}{e_{n2}} = \frac{-u_2 R_L}{(u_2+1)\, r_{p1} + r_{p2} + R_L + (u_2+1)\,(u_1+1)\, R_K}$$

To find $\frac{G_1}{G_2}$: (the factor by which the tube noise of V_2 is reduced when referred to the grid of V_1.)

$$\frac{G_1}{G_2} = \frac{\dfrac{-u_1(u_2+1)\,R_L}{(u_2+1)\,r_{p1} + r_{p2} + R_L + (u_2+1)(u_1+1)\,R_K}}{\dfrac{-u_2\,R_L}{(u_2+1)\,r_{p1} + r_{p2} + R_L + (u_2+1)(u_1+1)\,R_K}}$$

$$\frac{G_2}{G_1} = \frac{u_1\,(u_2+1)}{u_2} \approx u_1$$

Thus, the effect of the noise voltage of V_2 is reduced approximately by the factor $\frac{1}{u_1}$. The noise voltage due to the pentode stage is reduced by the factor G_1 when referred to the input grid. These reduction factors are sufficiently large to make the total equivalent input noise level, referred to the input, essentially equal to e_{n1}, the noise level of the lower triode stage.

Determination of Relationship Between Input Circuit Noise Voltage and Frequency Response for the General Case of an Input Transformer witha Capacitive Source Impedance.

A. Noise Relationships

The noise voltage appearing at any two terminals in a network may be calculated by the following expression:

$$(e_{noise})^2 = 4KTB_{eq}R(f)$$

where K = Boltzmann's Constant = 1.37×10^{-23}

T = Temperature, degrees Kelvin = 300° (assumed)

B_{eq} = Equivalent Bandwidth = 1 cps

R(f)= Real Part of Input Impedance

Thus, the calculation of noise voltage becomes essentially a calculation of the real part of the impedance seen across the two terminals.

The equivalent circuit of an input transformer with a capacitive source impedance may be represented as follows:

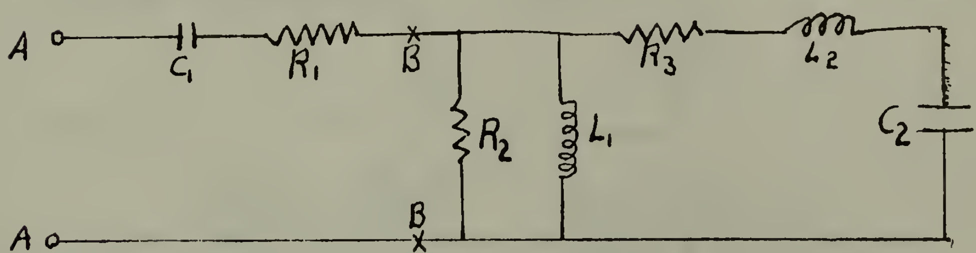


Fig. 20

C_1 = Source Capacitance

R_1 = Series damping R

R_2 = Shunt damping R

R_3 = Secondary Resistance referred to primary (assumed negligible)

L_1 = Primary L

L_2 = Leakage L referred to primary

C_2 = Shunt and Dist. C. referred to primary

$$\mathrm{Re}\left\{Z_{AA}\right\} = R_1 + \mathrm{Re}\left\{Z_{BB}\right\}$$

L_2 is usually small enough to be neglected at low frequencies. With $L_2 = 0$ (low freq.)

$$Z_{BB} = \frac{R_2 L_1 S}{L_1 S + R_2 + R_2 L_1 C_2 S^2}$$

$$\mathrm{Re}\left\{Z_{BB}\right\} = \frac{-R_2 L_1^2 S^2}{R_2(1 + L_1 C_2 S^2)^2 - L_1^2 S^2}$$

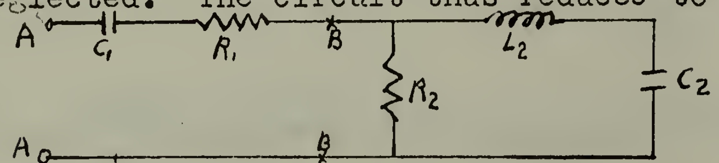
For $S = j\omega$,

$$\mathrm{Re}\left\{Z_{BB}\right\} = \frac{R_2 L_1^2 \omega^2}{R_2(1 - L_1 C_2 \omega^2)^2 + L_1^2 \omega^2}$$

Thus at Low Frequencies

$$e_n^2 = 4KT\left[R_1 + \frac{R_2 L_1^2 \omega^2}{\left[R_2(1 - L_1 C_2 \omega^2)\right]^2 + L_1^2 \omega^2}\right] \qquad \text{(Eqn. 1)}$$

At higher frequencies, L_1 (primary inductance) may usually be neglected. The circuit thus reduces to the following:



$$Z_{BB} = \frac{R_2\left(L_2 S + \frac{1}{C_2 S}\right)}{R_2 + L_2 S + \frac{1}{C_2 S}} = \frac{R_2(L_2 C_2 S^2 + 1)}{R_2 C_2 S + L_2 C_2 S^2 + 1}$$

$$\mathrm{Re}\left\{Z_{BB}\right\} = \frac{R_2\,(L_2 C_2 S^2 + 1)^2}{\left[L_2 C_2 S^2 + 1\right]^2 - (R_2 C_2 S)^2} = \frac{R_2}{1 - \frac{(R_2 C_2 S)^2}{(L_2 C_2 S^2 + 1)^2}}$$

For $S = j\omega$

$$\mathrm{Re}\left\{Z_{BB}\right\} = \frac{R_2}{1 + \frac{R_2^2 C_2^2 \omega^2}{(1 - L_2 C_2 \omega^2)^2}}$$

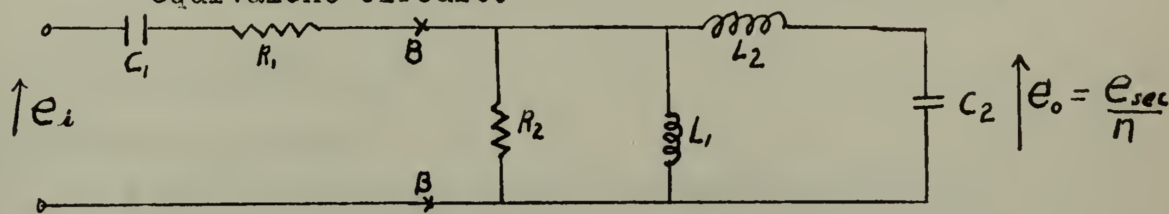
Thus at high frequencies:

$$\bar{e}_n^2 = 4KT\left[R_1 + \frac{R_2}{1 + \frac{R_2 C_2{}^2 \omega^2}{(1 - L_2 C_2 \omega^2)^2}}\right] \qquad \text{(Eqn. 2)}$$

The equations (1) and (2) show that the input circuit noise is determined as an explicit function of all the circuit parameters except C_1. These circuit parameters are uniquely determined by the specification of the frequency response required and the source capacitance, C_1. In this manner, C_1 and the frequency response, once prescribed, determine the input noise level of the circuit. We now attempt to correlate the circuit parameters from the specified frequency response and source capacitance.

B.) Frequency Response Relationships

1. Determination of the transfer function of the equivalent circuit.



$$Z_{eq} = Z_{BB} = \frac{1}{\frac{1}{R_2} + \frac{1}{L_1 S} + \frac{1}{L_2 S + \frac{1}{C_2 S}}} = \frac{1}{\frac{1}{R_2} + \frac{1}{L_1 S} + \frac{C_2 S}{L_2 C_2 S^2 + 1}}$$

$$Z_{BB} = \frac{R_2 L_1 S \quad (L_2 C_2 S^2 + 1)}{L_1 S\,(L_2 C_2 S^2 + 1) + R_2 (L_2 C_2 S^2 + 1) + R_2 L_1 C_2 S^2}$$

$$\frac{e_o}{e_1} = H(s) = \frac{Z_{BB} \frac{1}{(L_2 C_2 S^2 + 1)}}{\frac{1}{C_1 S} + R_1 + Z_{BB}} = \frac{C_1 S Z_{BB} \left(\frac{1}{L_2 C_2 S^2 + 1}\right)}{1 + R_1 C_1 S + C_1 S\, Z_{BB}}$$

Let denominator of $Z_{BB} = D$

$$H\,(s) = \frac{\frac{C_1 R_2 L\, S^2\, (L_2 C_2 S^2 + 1)}{D} \quad \frac{1}{(L_2 C_2 S^2 + 1)}}{1 + R_1 C_1 S + \frac{C_1 L_1 R_2 S^2 \,[L_2 C_2 S^2 + 1]}{D}}$$

$$H(s) = \frac{C_1R_2L_1S^2}{D + R_1C_1SD + R_2C_1L_1S^2\left(L_2C_2S^2+1\right)}$$

$D =$ Denominator of Z_{BB}

$$= L_1 + L_2C_2S^3 + L_1S + R_2L_2C_2S^2 + R_2 + R_2L_1C_2S^2$$

$$= S^3\left(L_1L_2C_2\right) + S^2\left(R_2L_2C_2+R_2L_1C_2\right) + L_1S+R_2$$

Denominator of $H(s) = D + R_1C_1SD + R_2C_1L_1L_2C_2S^4 + R_2C_1L_1S^2$

Denominator of $H(s)$:

$$= S^3(L_1L_2C_2)+S^2R_2C_2(L_1+L_2)+L_1S\ R_2$$

$$+ S^4\ R_1C_1L_1L_2C_2+S^3R_1C_1(R_2L_2C_2+R_2L_1C_2)+R_1C_1L_1S^2+R_1R_2C_1S$$

$$+ S^4R_2C_1L_1L_2C_2+R_2C_1L_1S^2$$

Denominator of $H(s)$:

$$= S^4(R_1C_1L_1L_2C_2+R_2C_1L_1L_2C_2) = S^4(R_1+R_2)L_1C_1L_2C_2$$

$$+ S^3\left[L_1L_2C_2+R_1C_1(R_2L_2C_2+R_2L_1C_2)\right] = S^3\left[L_1L_2C_2+ R_1C_1R_2C_2(L_1+L_2)\right]$$

$$+ S^2\left[R_2C_2(L_1+L_2)+R_1C_1L_1+R_2C_2L_1\right] = S^2\left[R_2C_2(L_1+L_2)+R_1C_1L_1+R_2C_1L\right]$$

$$+ S\ (L_1+R_1R_2C_1) \qquad = S\ (L_1+R_1R_2C_1)$$

$$+ R_2 \qquad = R_2$$

Dividing numerator and denominator of $H(s)$ by the coefficient of S^4, we get:

$$\text{Numerator of } H(s) = \frac{R_2L_1C_1S^2}{(R_1+R_2)L_1C_1L_2C_2} = \frac{R_2S^2}{(R_1+R_2)L_2C_2}$$

Denominator of $H(s)$ =

$$= S^4 \qquad = S^4$$

$$+ \frac{S^3\left[L_1L_2C_2+R_1C_1R_2C_2(L_1+L_2)\right]}{L_1C_1L_2C_2(R_1+R_2)} = \frac{S^3\ L_1L_2+R_1R_2C_1(L_1+L_2)}{L_1L_2C_1(R_1+R_2)}$$

$$+ \frac{S^2\left[R_2C_2(L_1+L_2)+R_1C_1L_1+R_2C_1L_1\right]}{L_1C_1L_2C_2(R_1+R_2)} = \frac{S^2\ R_2C_2(L_1+L_2)+R_1C_1L_1+R_2C_1L_1}{L_1L_2C_1C_2(R_1+R_2)}$$

$$+ \frac{S(L_1+R_1R_2C_1)}{L_1L_2C_1C_2(R_1+R_2)} + \frac{R_2}{L_1L_2C_1C_2(R_1+R_2)}$$

Finally,

$$H(s) = \frac{\dfrac{R_2 S^2}{(R_1+R_2)L_2C_2}}{S^4 + S^3\,\dfrac{L_1L_2+R_1R_2C_1(L_1+L_2)}{L_1L_2C_1\;R_1+R_2} + S^2\,\dfrac{R_2C_2(L_1+L_2)+L_1C_1(R_1+R_2)}{L_1L_2C_1C_2\;R_1+R_2} + S\,\dfrac{L_1+R_1R_2C_1}{L_1L_2C_1C_2(R_1+R_2)} + \dfrac{R_2}{L_1L_2C_1C_2(R_1+R_2)}} \qquad (Eq.\ 3)$$

2.) <u>Correlation of Transfer Function with specified Freq. Response.</u>

If we assume the frequency response to be specified as shown in Fig. 21.

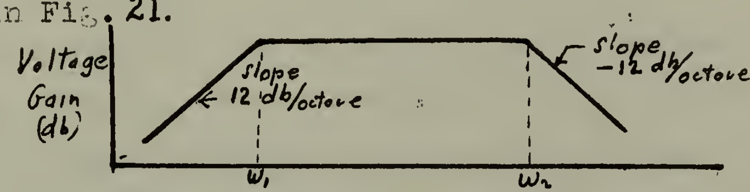


Fig. 21. Typical Response

the transfer function must be of the form:

$$H(s) = K\,\frac{S^2}{(S+\omega_1)^2(S+\omega_2)^2} = K\,\frac{S^2}{[S^2+2\omega_1S+\omega_1^2][S^2+2\omega_2S+\omega_2^2]}$$

$$H(s) = K\,\frac{S^2}{S^4+S^3(2\omega_1+2\omega_2)+S^2(\omega_1^2+4\omega_1\omega_2+\omega_2^2)+S(2\omega_2\omega_1^2+2\omega_1\omega_2^2)+\omega_1^2\omega_2^2} \qquad (Eq.\ 4)$$

Correlation between the transfer function of the transformer circuit with the specified transfer function may be obtained by equating the coefficients of like powers of S in the denominators of Equations (3) and (4).

There will result four simultaneous equations, each equation describing a constraint on the values of the dependent variables, (R_1, R_2, C_2, L_1) These are:

$$\frac{L_1L_2+R_1R_2C_1(L_1+L_2)}{L_1L_2C_1(R_1+R_2)} = 2\omega_1+2\omega_2 \qquad \text{(Eqn 5)}$$

$$\frac{R_2C_2(L_1+L_2)+L_1C_1(R_1+R_2)}{L_1L_2C_1C_2(R_1+R_2)} = \omega_1^2+4\omega_1\omega_2+\omega_2^2 \qquad \text{(Eqn 6)}$$

$$\frac{L_1+R_1R_2C_1}{L_1L_2C_1C_2(R_1+R_2)} = 2\omega_2\omega_1^2+2\omega_1\omega_2^2 \qquad \text{(Eqn. 7.)}$$

$$\frac{R_2}{L_1L_2C_1C_2(R_1+R_2)} = \omega_1^2\omega_2^2 \qquad \text{(Eqn. 8)}$$

In this set of four equations, the independent variables are ω_1, ω_2, and C_1. L_2 (leakage inductance) is a function of the primary inductance and may be related to it by a constant, depending on the type of winding and method of construction.

If we specify the turns ratio, ω_1, ω_2 and C_1, these four equations are solvable for the values of R_1, R_2, L_1, and C_2. Substitution of these values into the thermal noise voltage equations (1) and (2), will determine the spectrum noise level resulting from a specified frequency response and source capacitance. Comparison of this level with the required level at the critical frequency will indicate whether or not it is possible to determine the frequency response in the transformer circuit.

<u>Simplified Case</u>

While the above procedure is theoretically possible, the solution of the four simultaneous equations is not easily accomplished because of their non-linearity. Because of the labor involved, this method may be impracticable in actual use. In certain cases where simplifying assumptions are justified, however, a quick approximate solution may be obtained.

If we assume the following:

$$\omega_2 \gg \omega_1$$

$$R_2 \gg R_1$$

$$C_1 \gg C_2$$

the low frequency equivalent circuit is as shown in Fig. 22

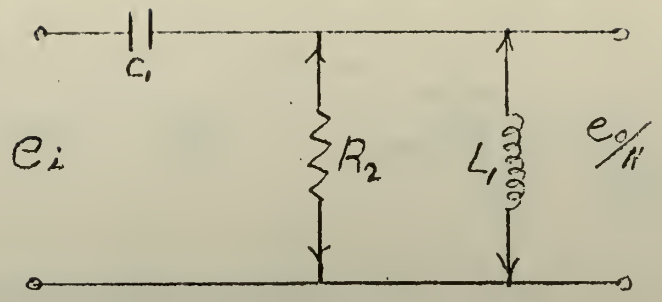


Fig. 22

The transfer function H(s), becomes:

$$H(s) = \frac{e_o/n}{e_i} = \frac{S^2}{S^2 + \frac{S}{R_2C_1} + \frac{1}{L_1C_1}}$$

The denominator factors into:

$$S = -\frac{1}{2R_2C_1} \pm \sqrt{\frac{1}{4R_2{}^2C_1{}^2} - \frac{1}{L_1C_1}}$$

Critical damping will achieve the desired low frequency response.

For critical damping:

$$\omega_1{}^2 = \frac{1}{L_1C_1} = \frac{1}{4R_2{}^2C_1{}^2}$$

Then $L_1 = \frac{1}{\omega_1{}^2C_1}$ and $R_2 = \frac{1}{2\omega_1C_1}$. (Eqn. 9)

The high frequency equivalent circuit simplifies to that of Fig. 23 .

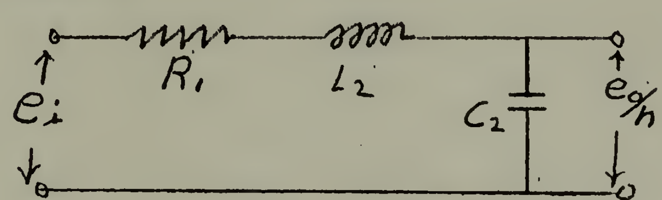
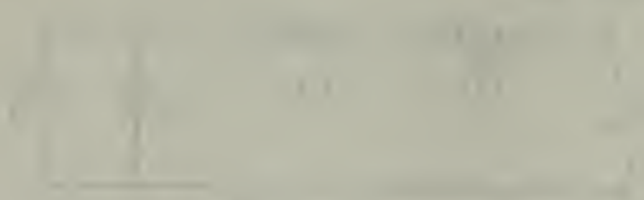


Fig. 23.

We further assume that L_2 is a known function of L_1 (depending on transformer construction) and is then



determined once L_1 has been determined.

The transfer function:

$$H(s) = \frac{eo/n}{e_i} = \frac{\frac{1}{L_2C_2}}{S^2 + \frac{R_1}{L_2} S + \frac{1}{L_2C_2}}$$

For critical damping:

$$\omega_2^2 = \frac{1}{L_2C_2} = \frac{R_1^2}{4L_2^2}$$

$$\text{and } R_1 = 2\omega_2 L_2 \quad \text{and} \quad C_2 = \frac{1}{\omega_2^2 L_2} \qquad \text{(Eqn. 10)}$$

This presents a rapid means of approximating R_1, R_2, L_1, and C_2 for specified values of ω_1, ω_2, and C_1. Substitution of these values as given by equations (9) and (10) into equations (1) and (2) will yield the theoretical value of noise level of the input transformer.

Output Filter Investigation

In attempting to shape the high end frequency response in the transformer circuit (see Appendix page 37.), two disadvantages result. R_1, the series damping resistor, contributes noise at all frequencies, and C_2, the added shunt C necessary to move the resonant peak down to 20 kc. constitutes, with source capacitance C_1, a voltage divider which lowers transformer gain. The circuit is given in Fig. 1.

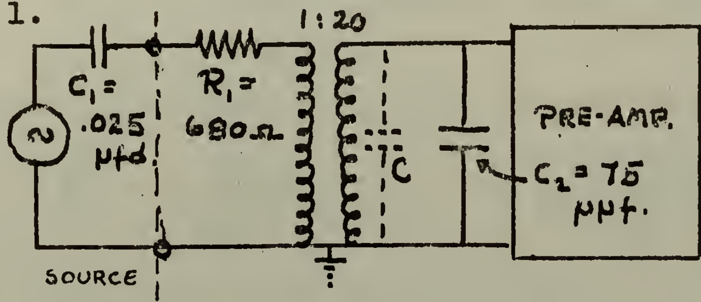


Fig. 1

The transformer parameters, referred to the primary are:

L_L, Leakage Inductance = 4 mh

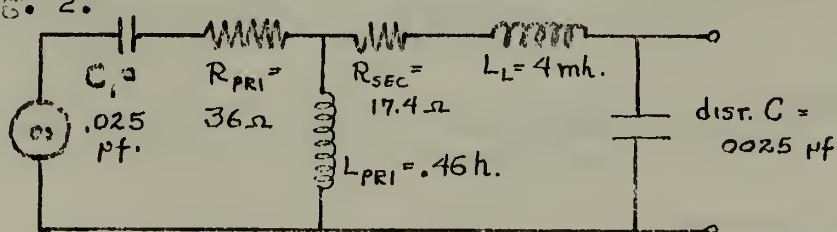
L_P, Primary Inductance = 0.46

R, Primary and Secondary= 53.4 ohms
resistance

C, Secondary distributed
shunt C = .0025 uf

Transformer Equivalent Circuit

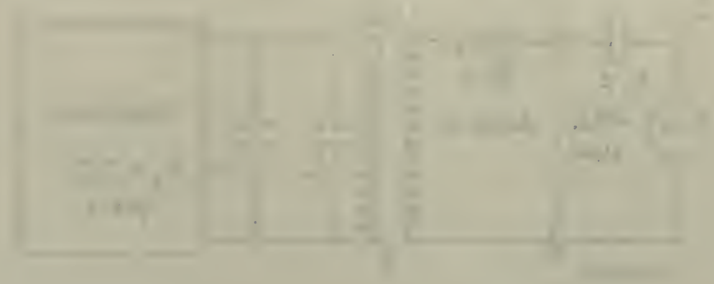
If we remove R_1 and C_2, we have the circuit shown in Fig. 2.

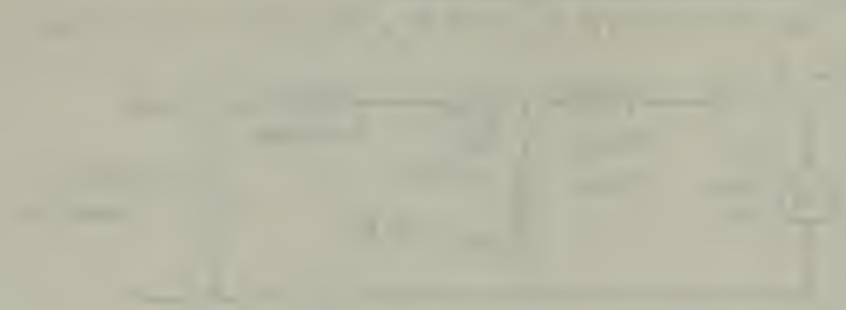


All parameter values referred to primary

Fig. 2 Transformer Equivalent Circuit.

Page 69a, Appendix, shows the frequency response of this circuit. (Curve A).





At high frequencies, the resonant peak occurs at 50 kc. If we assume that at high frequencies the primary inductance, L_p, becomes an open circuit, and the source capacitance C_1 becomes a short circuit, the equivalent circuit simplifies to the one given in Figure 3.

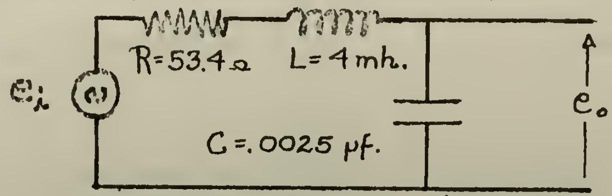


Fig. 3. High frequency transformer equivalent circuit.

The transfer function of this circuit is:

$$\frac{e_o}{e_i} = \frac{\frac{1}{Cs}}{\frac{1}{Cs}+LS+R} = \frac{\frac{1}{LC}}{S^2 + \frac{RS}{L} + \frac{1}{LC}} \qquad \text{(Eqn. 1)}$$

The roots of the denominator are:

$$S = -\frac{R}{2L} \pm j\sqrt{\frac{1}{LC} - \left(\frac{R}{2L}\right)^2}$$

and represents a conjugate pole pair on the complex frequency (S) plane. The location of these poles is established as follows:

$$\omega_n = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{4X10^{-3}X25X10^{-10}}} = 3.16 \text{ X } 10^5 \text{ rad./sec.}$$

$$f_n = \frac{\omega_n}{2\pi} = \frac{3.16 \text{ X } 10^5}{6.28} = 50.3 \text{ kc, agreeing closely}$$

with the observed resonant peak at 50 kc. (Appendix pg.69a)

$$\text{Real Part, } \sigma = \frac{R}{2L} = \frac{53.4}{2(4X10^{-3})} = 6.67 \text{ X } 10^3$$

$$\text{Damping ratio, } \zeta = \frac{\frac{R}{2L}}{\omega_n} = \frac{6.67X10^3}{3.16X10^5} = 2.11X10^{-2}$$

$$\omega_d = \omega_n\sqrt{1-\zeta^2} = \omega_n\sqrt{1-(2.11X10^{-2})^2}$$

Therefore, $\omega_d \approx \omega_n = 3.16 \times 10^5$ rad/sec., and the poles occur at $S=-\sigma \pm j\omega_n = -6.67 \times 10^3 \pm j3.16 \times 10^5$

They are shown pictorially in Fig. 4.

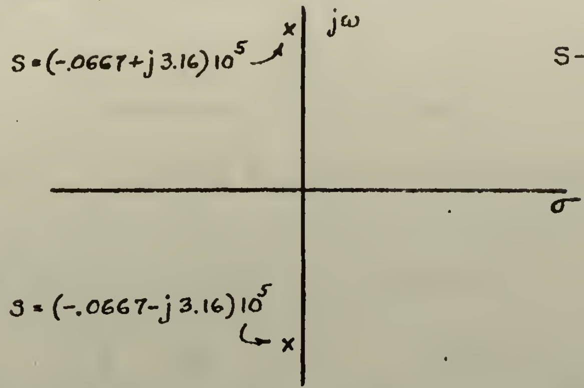


Fig. 4. Pole-Zero Pattern of Transformer at High Frequencies.

For the required frequency response breaking at 20 kc, and falling off at a median slope of 12 db/octave, we need a double-order real axis pole as shown in Fig. 5.

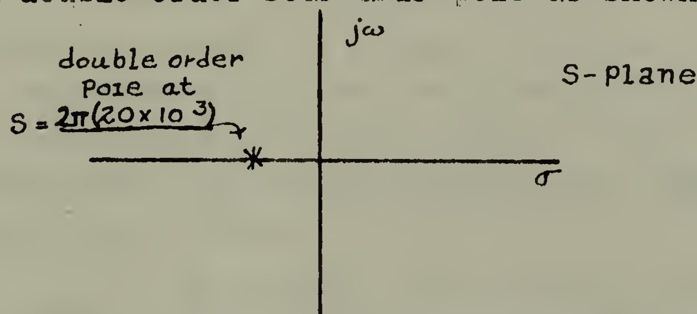


Fig. 5 Desired High Frequency Pole Zero Pattern.

<u>Design of the Output Filter</u>

The purpose of the output filter, when cascaded with the transformer, is to change the pole-zero pattern from that of Fig. 4 to that of Fig. 5, thus eliminating the resonant peak and giving the desired roll-off. With respect to frequency response, the input transformer and the output filter are in cascade since the pre-amplifier itself has a perfectly flat response to well above the frequencies of interest.

In view of the miniaturization requirement, a simple filter configuration must be used.

The one selected is shown in Fig. 6.

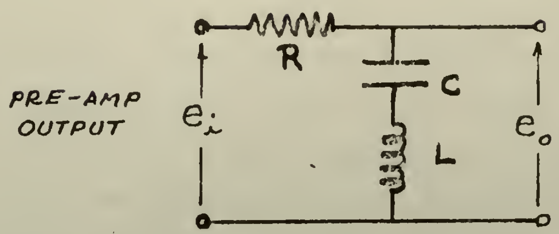


Fig. 6. Output Filter Configuration

The filter transfer function is:

$$\frac{e_o}{e_i} = \frac{\frac{1}{CS} + LS}{\frac{1}{CS} + LS + R} = \frac{S^2 + \frac{1}{LC}}{S^2 + \frac{RS}{L} + \frac{1}{LC}}$$

$$H(s) = \frac{e_o}{e_i} = \frac{\left(S + j\frac{1}{\sqrt{LC}}\right)\left(S - j\frac{1}{\sqrt{LC}}\right)}{\left[S + \frac{R}{2L} + j\sqrt{\frac{1}{LC} - \left(\frac{R}{2L}\right)^2}\right]\left[S + \frac{R}{2L}\, j\sqrt{\frac{1}{LC} - \left(\frac{R}{2L}\right)^2}\right]} \qquad \text{(Eqn. 2)}$$

Where the numerator factors represent zeros of H(s) and denominator factors are poles of H(s).

To obtain the pole-zero pattern of Fig. 5, the filter parameters must simultaneously meet three requirements, as follows:

A. $\frac{1}{\sqrt{LC}} = 50\ \text{kc} \times (2\pi)$ to provide series resonance and maximum attenuation at 50 kc. to counteract the transformer resonant peak at this frequency.

B. $\frac{R}{2L} = 20\ \text{kc} \times (2\pi)$ to establish the upper break frequency at 20 kc.

C. $\frac{1}{\sqrt{LC}} = \frac{R}{2L}$ to make the radical terms zero and thus avoid a conjugate pole pair due to the filter and giving rise to another resonance.

These three requirements cannot all be met with a simple R-L-C network. The approach is therefore first to cancel (as nearly as possible) the transformer poles which produce the high end peak, by the zeros of the

filter. Since we cannot achieve the desired double order real-axis pole, we may get a suitable roll-off in the neighborhood of 20 kc. by allowing these poles to split apart, keeping one near its σ value for 20 kc, and letting the other go to a much higher σ value. In doing this, we are taking advantage of the latitude of the specified response curve, i.e., roll-off between -6 and -18 db/octave (Appendix pg. 69a., Curve B.)

Following this reasoning, the value of L and C are determined by the 50 kc. resonant frequency and R is selected to give slight overdamping.(rather than the desired, but not obtainable, critical damping). The resulting pole-zero pattern of the filter, super-imposed of the transformer pattern is given in Fig. 7.

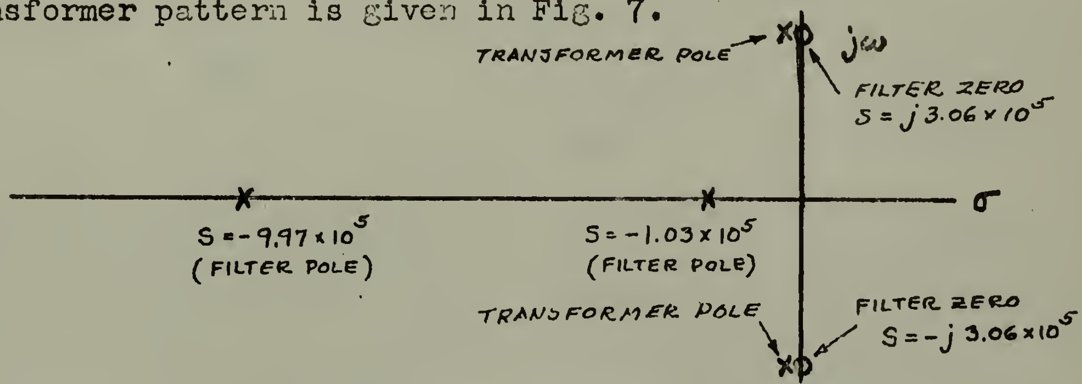
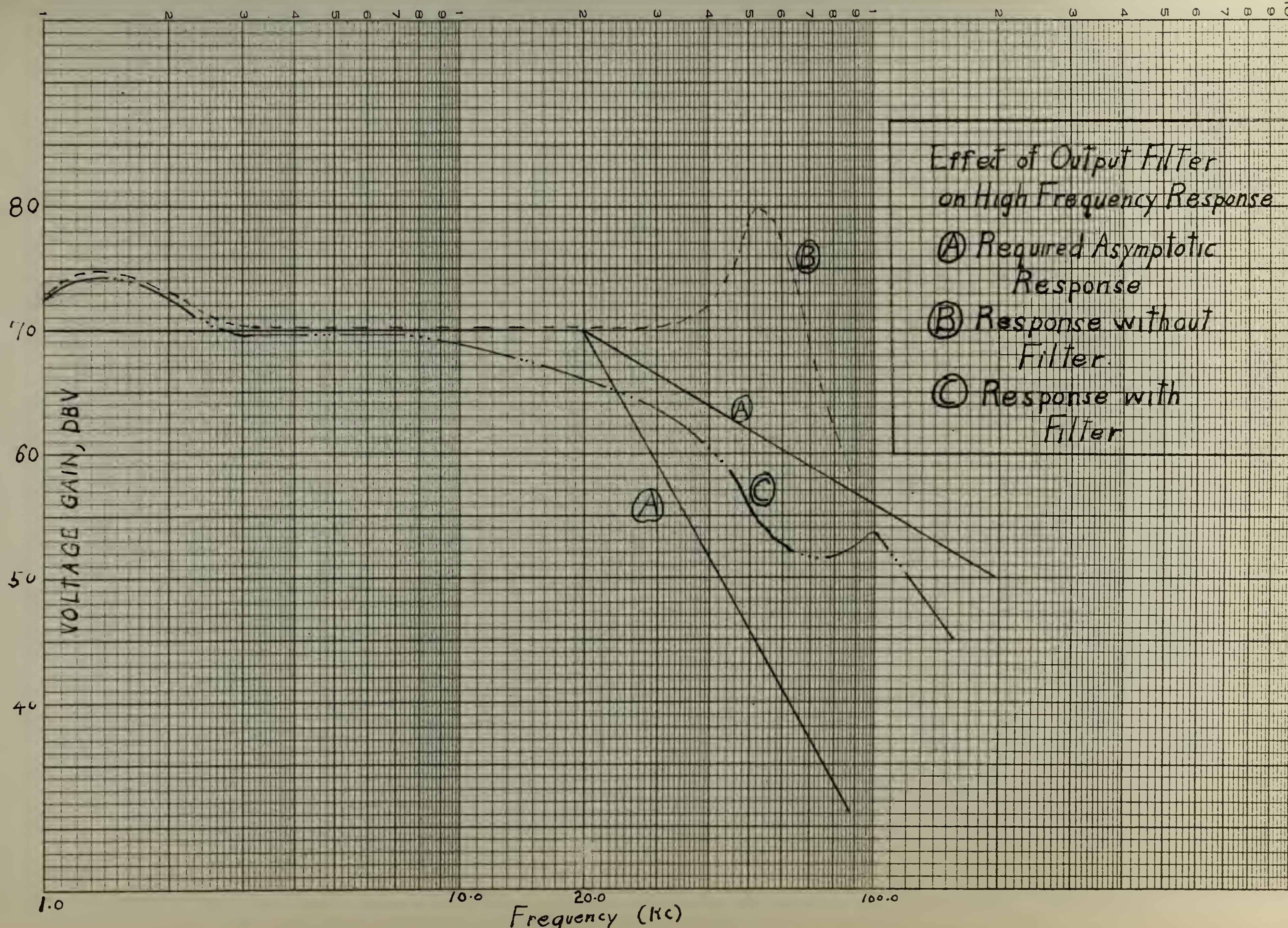


Fig. 7 - Combined pole-zero pattern of transformer and output filter.

The resulting frequency response is shown on Curve C, Appendix pg.69a. It is seen to lie within the specified limits.



Effect of Output Filter on High Frequency Response
(A) Required Asymptotic Response
(B) Response without Filter
(C) Response with Filter
A
B
C
VOLTAGE GAIN, DBV
80
70
60
50
40
1.0
10.0
20.0
100.0
Frequency (Kc)

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SYMBOLS

B or B_{eq}	-	Equivalent Bandwidth
e	-	Incremental voltage
e	-	Electron charge (1.59×10^{-19}) coulcombs)
e_n	-	Noise voltage (root mean square value)
G	-	Voltage Gain
gm	-	Transconductance
H (s)	-	Transfer Function relating output voltage to input voltage
i_n	-	Noise current (root mean square value)
j	-	$\sqrt{-1}$
K	-	Boltzmann's Constant (1.37×10^{-23} Joules per degree Kelvin)
r_p	-	Dynamic plate resistance
s	-	Complex operator of form $\sigma + j\omega$
T	-	Temperature (degrees Kelvin)
Z	-	Impedance
ω	-	Frequency (radians per second)
u	-	Amplification factor

Thesis 28793
K33 Kendall
Development of a miniaturized, low noise sonar pre-amplifier.